PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES
INDIA

Part 2] May, 1937 [Volume 7

THE MATHEMATICAL THEORY OF A NEW RELATIVITY

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Received July 8, 1937.

CHAPTER XV

The Rotational Theory of Light and Matter

SECTION I

Introduction

I. "There is something radically wrong with the present fundamental conceptions of Physics, and we do not see how to set it right." (Eddington)[1] Light and matter show at the same time both the particle and the wave aspects—an apparent contradiction. As the true picture can not be visualised, it has in despair been completely discarded. Physics is now dominated by purely artificial mathematical devices, which seem to work well, without disclosing their real Physical significance. When one tries to form a mental conception, there is nothing but confusion and chaos. Everything consists of waves, stretching from infinity to infinity; yet they are not real waves, not even waves of energy, but a mere mathematical fiction. They are "waves of probability or chance." Jeans would regard these waves as consisting of "knowledge". Max Born thinks "it would be better to call them waves of partial knowledge". Unkind critics call them "waves of ignorance". As matters stand, all such descriptions

are equally appropriate. Wave Mechanics evades the questions how energy is propagated through space without matter or medium, why the waves do not scatter away or how an infinite train of waves can suddenly converge to a point when meeting another such train ; nor has it yet at all tackled numerous other problems of Physics including polarisation. De Broglie wave associated with the electron remains mysterious and obscure. "Unfortunately, however, it (Wave Mechanics) compels one to use a continuum the number of whose dimensions is not that ascribed to Physics hitherto (four) but rises indefinitely with the number of particles constituting the system under consideration" (Einstein),[2] being three times as many. It is an irony of fate that Relativity in order to explain the interaction of a million stars should require a space of four million dimensions. Wave Mechanics using imaginary operators and abounding in $\sqrt{-1}$ causes bewilderment.

2. I have already ventured to put forward a Rotational Theory of Light essentially corpuscular in character. Light consists of swarms of discreet units, Newton's corpuscles, now called radions, with one addition that in its inner structure a radion consists of two components of equal and opposite charges (found to be of equal masses also) rotating round their common centre of gravity, which is moving forward with a uniform velocity. The translational and the rotational motions give it both the particle and wave aspects simultaneously e. g. both momentum and wavelength. Light then becomes an electromagnetic system, creating and carrying its field with it. Maxwell's equations follow naturally. It has been shown that such a system automatically propagates itself with the known velocity of light.

3. Further developments are offered in this Chapter. The orbit, referred to the moving axes, may not be perpendicular but may be inclined to the path of motion. A hydrogen atom, consisting of a positively charged proton and a negatively charged electron, has a similar structure. Rotational mechanics applies to it in a similar way, only the masses are unequal and the orbit is generally elliptical. The law u. $v = c^2$ follows from this rotational cum translational motion. De Broglie's assumption p. $\lambda = h$ is then deduced. Einstien's formula $E = mc^2$ is explained. Planck's assumption $E = h\nu$ is also deduced. Heisenberg – Schrodinger's equation

$$\left[\nabla^2 + \frac{8\pi^2 m}{h^2}(E - V)\right]\psi = 0$$

is easily obtained, with other necessary results. Lorentz transformations utilised in Relativity are explained. Thus many mysteries are solved, as the picture is clearly before the mind. Some further developments, reserved for a later paper, will be interesting.

4. For the solution of Maxwell's equations when the E's are expressed in exponentials I express my indebtedness to Dr. N. S. Japolsky, whose method and results are acknowledged in Section III headed as "Japolsky's Solutions". For light, I have added a simpler Physical solution.

There is undoubtedly some resemblance between his rotating electromagnetic waves or "whirls" stretching from infinity to infinity, or the whole train either bodily moving along the axis of symmetry or rotating round it, and my rotatary binary system moving forward from the source to the obstacle, carrying its own electromagnetic field with it. There are many common features between the two, and part of the mathematics certainly seems to be common too. But there are also a few essential differences:—

(1) The Physical nature of the whirls is not known; while radions are like binary stars, revolving and moving forward.

(2) A rotating wave or whirl requires a medium for its propagation, and a fluid medium too; it is awkward to fill the whole space with fluid medium in order to find means for light to come from any single star. A radion can travel by itself through the void space; indeed a medium would obstruct it.

(3) A whirl may well expand and spread away; so some assumption has to be made for its retaining form and size during its passage through vast space. On the other hand, no such assumption as to the stability of a binary system is required, for the mutual attraction of the components will maintain their orbit.

(4) A whirl should create more and more whirls round itself in the medium and its energy would soon be scattered away. An isolated radion conserves its energy; until it meets an obstruction, it gets no chance for parting with any energy.

(5) It is not clear why a whirl of proton or electron should possess a characteristic mass. Another difficulty is that such energy or mass would have to be distributed all along its infinite length and singularities should not occur. No such difficulty arises in the case of discreet radionic units.

(6) It is difficult to see what happens when two whirls collide. If they are superposed, they should pass through. If not, then after collision the two infinite trains must scatter away ; but it is difficult to conceive of a Compton scattering of two infinite waves approaching each other from opposite directions along the same axis. Radions can of course collide like billiard balls.

(7) It is doubtful whether without any charged particles forces acting on the field can at all be produced.[3] The charged binary system will of course produce a field.

But these difficulties are not peculiar to Japolsky's "whirls". They are inherent in every system of waves, including those of Wave Mechanics.

5. I again express my deep gratitude to Dr. D. S. Kothari, M. Sc., Ph. D., Reader in Physics at the Delhi University and Mr. Ram Niwas Rai, M. Sc., Lecturer in Physics at the Allahabad University for their great kindness in making several valuable suggestions.

SECTION II

Electromagnetic Equations

Let E and M denote electric and magnetic intensities and suffixes indicate their components. Then Maxwell's electromagnetic equations in cylindrical coordinates, as given in my Rotational Theory of Light[4] for a region free from charges are :—

$$\left.\begin{aligned} \frac{k}{c}\frac{\partial E_r}{\partial t} &= \frac{1}{r}\frac{\partial M_z}{\partial \theta} - \frac{\partial M_\theta}{\partial z} \\ \frac{k}{c}\frac{\partial E_\theta}{\partial t} &= \frac{\partial M_r}{\partial z} - \frac{\partial M_z}{\partial r} \\ \frac{k}{c}\frac{\partial E_z}{\partial t} &= \frac{1}{r}\frac{\partial (rM_\theta)}{\partial r} - \frac{1}{r}\frac{\partial M_r}{\partial \theta} \end{aligned}\right\} \quad \left.\begin{aligned} -\frac{\mu}{c}\frac{\partial M_r}{\partial t} &= \frac{1}{r}\frac{\partial E_z}{\partial \theta} - \frac{\partial E_\theta}{\partial z} \\ -\frac{\mu}{c}\frac{\partial M_\theta}{dt} &= \frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} \\ -\frac{\mu}{c}\frac{\partial M_z}{\partial t} &= \frac{1}{r}\frac{\partial (rE_\theta)}{\partial r} - \frac{1}{r}\frac{\partial E_r}{\partial \theta} \end{aligned}\right\}$$

$$\left.\begin{aligned} \frac{1}{r}\frac{\partial}{\partial r}(rE_r) + \frac{1}{r}\frac{\partial E_\theta}{\partial \theta} + \frac{\partial E_r}{\partial z} &= 0 \\ \frac{1}{r}\frac{\partial}{r}(rM_r) + \frac{1}{r}\frac{\partial M_\theta}{\partial \theta} + \frac{\partial M_z}{\partial z} &= 0 \end{aligned}\right\} \quad \ldots \quad (43{\cdot}1)$$

Proceeding as in Ch. XIV Sec. V, we differentiate the first group with respect to t, eliminate the magnetic components by substitution from the second group, and get

$$\left.\begin{aligned}
\frac{\mu k}{c^2}\frac{\partial^2 E_r}{\partial t^2} &= -\frac{1}{r^2}\frac{\partial^2 (rE\theta)}{\partial r\partial\theta} + \frac{1}{r^2}\frac{\partial^2 E_r}{\partial\theta^2} + \frac{\partial^2 E_z}{\partial z^2} - \frac{\partial^2 E_z}{\partial r\partial z}\\
\frac{\mu k}{c^2}\frac{\partial^2 E\theta}{\partial t^2} &= -\frac{1}{r}\frac{\partial^2 E_z}{\partial z\partial\theta} + \frac{\partial^2 E\theta}{\partial z^2} + \frac{\partial}{\partial r}\left(\frac{\partial (r\,E\theta)}{r\partial r}\right) - \frac{\partial}{\partial r}\left(\frac{\partial E_r}{r\partial\theta}\right)\\
\frac{\mu k}{c^2}\frac{\partial^2 E_z}{\partial t^2} &= -\frac{\partial}{r\partial r}\left(r\frac{\partial E_r}{\partial z}\right) + \frac{\partial}{r\partial r}\left(r\frac{\partial E_z}{\partial r}\right) + \frac{\partial^2 E_z}{r^2\partial\theta^2} - \frac{\partial^2 E\theta}{r\partial\theta dz}\\
\text{Also}\quad \frac{1}{r}\frac{\partial}{\partial r}(r\,E_r) &+ \frac{1}{r}\frac{\partial E\theta}{\partial\theta} + \frac{\partial E_z}{\partial z} = 0
\end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (43{\cdot}2)$$

Similarly we obtain,

$$\left.\begin{aligned}
\frac{\mu k}{c^2}\frac{\partial^2 M_r}{\partial t^2} &= -\frac{1}{r^2}\frac{\partial^2 (r\,M\theta)}{\partial r\partial\theta} + \frac{1}{r^2}\frac{\partial^2 M_r}{\partial\theta^2} + \frac{\partial^2 M_r}{\partial z^2} - \frac{\partial^2 M_z}{\partial r\partial z}\\
\frac{\mu k}{c^2}\frac{\partial^2 M\theta}{\partial t^2} &= -\frac{1}{r}\frac{\partial^2 M_z}{\partial z\partial\theta} + \frac{\partial^2 M\theta}{\partial z^2} + \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial (rM\theta)}{\partial r}\right] - \frac{\partial}{\partial r}\left(\frac{\partial M_r}{r\partial\theta}\right)\\
\frac{\mu k}{c^2}\frac{\partial^2 M_z}{\partial t^2} &= -\frac{\partial}{r\partial r}\left(r\frac{\partial M_r}{\partial z}\right) + \frac{\partial}{r\partial r}\left(r\frac{\partial M_z}{\partial r}\right) + \frac{\partial^2 M_z}{r^2\partial\theta^2} - \frac{\partial^2 M\theta}{r\partial\theta\partial z}\\
\frac{1}{r}\frac{\partial}{\partial r}(r\,M_r) &+ \frac{1}{r}\frac{\partial M\theta}{\partial\theta} + \frac{\partial M_z}{\partial z} = 0
\end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (43{\cdot}3)$$

SECTION III

Japolsky's Solutions

1. N. S. Japolsky[5] has considered the case where the components of the electromagnetic field are simple harmonic functions of t, z and θ and can be expressed as $E.\, e^{i(wt + \kappa z + n\theta)}$ where the coefficients may be complex. He has in a most elegant manner obtained the solutions of Maxwell's equations by transforming them into Bessel's equations. For the purposes of this paper it will be quite sufficient to find the expression for E_z. The others will be merely reproduced.

Let $E_r = R e^{i\phi}$; $E_\theta = \theta e^{i\phi}$; $E_z = Z e^{i\phi}$,

where R, θ, Z are functions of r only and $\phi = wt + \kappa z + n\theta$. Substituting these values in the first three equations of (43·2), and putting

$c_1 = \frac{c}{\sqrt{\mu k}}$ we get

$$-\frac{w^2}{c_1^2}R = -\frac{in}{r^2}\frac{\partial(r\theta)}{\partial r} - \frac{n^2}{r^2}R - \kappa^2 R - i\kappa\frac{\partial Z}{\partial r}.$$

$$-\frac{w^2}{c_1^2}\theta = \frac{n\kappa}{r}Z - \kappa^2\theta + \frac{\partial}{\partial r}\left[\frac{\partial(r\theta)}{r\partial r}\right] - in\frac{\partial}{\partial r}\left(\frac{R}{r}\right)$$

$$-\frac{w^2}{c_1^2}Z = -i\kappa\frac{\partial}{r\partial r}(rR) + \frac{\partial}{r\partial r}\left(r\frac{\partial Z}{\partial r}\right) - \frac{n^2}{r^2}Z + \frac{n\kappa}{r}\theta. \quad \ldots \quad (44·1)$$

We multiply the first equation by $-ir$ and then differentiate with respect to r. We multiply the second equation by n. We multiply the third equation by κ r. If we then add up the three results we find that all the terms on the right hand side cancel one another. Therefore omitting the common factor $-\frac{w^2}{c_1^2}$, we have,

$$\kappa r Z - i\frac{\partial}{\partial r}(rR) + n\theta = 0 \quad \ldots \quad \ldots \quad (44·2)$$

We can now eliminate R and θ by multiplying (44·2) by $\frac{\kappa}{r}$ and adding it to the last equation of (44·1), and get,

$$\frac{\partial}{r\partial r}\left(r\frac{\partial Z}{\partial r}\right) + \left(\frac{w^2}{c_1^2} - \kappa^2 - \frac{n^2}{r^2}\right)Z = 0 \quad \ldots \quad \ldots \quad (44·3)$$

Substituting $\rho = \sqrt{\frac{w^2}{c_1^2} - \kappa^2}\,.\,r$ and working out the differentiation of the first term of (44·3) and also substituting total differentials because R, θ, Z are functions of r only we get,

$$\left[\frac{d^2}{d\rho^2} + \frac{d}{\rho d\rho} + \left(1 - \frac{n^2}{\rho^2}\right)\right]Z = 0 \quad \ldots \quad \ldots \quad (44·4)$$

The above equation is a well-known Bessel equation whose solution, that is finite throughout the space, is

$$Z = A\, J_n(\rho), \text{ where } \rho = \sqrt{\frac{w^2}{c_1^2} - \kappa^2}.\, r,$$

and $J_n(\rho)$ is a Bessel coefficient of the order n of the variable ρ ... (44·5) For real waves, the independent variable ρ should be real otherwise the amplitude will tend to ∞ when r approaches ∞.

He has by analogous transformations also obtained the expressions for the other quantities.

$$\left.\begin{aligned} i\,R &= -A_1 J_{n+1} + A_2 J_{n-1} \\ \theta &= A_1 J_{n+1} + A_2 J_{n-1} \end{aligned}\right\} \quad \ldots \quad \ldots \quad (44\cdot6)$$

where however $A = -\sqrt{\frac{w^2}{c_1^2} - \kappa^2}.\frac{A_1 + A_2}{\kappa}$.

Also if the magnetic forces be $M_r = R'.\, e^{i(wt + \kappa z + n\theta)}$ etc,

then

$$\left.\begin{aligned} Z' &= i\sqrt{\frac{w^2}{c_1^2} - \kappa^2}.\,(A'_1 + A'_2)\, J_n \\ R' &= -A_1' J_{n+1} + A'_2 J_{n-1} \\ \theta' &= i\,(A_1' J_{n+1} + A'_2 J_{n-1} \end{aligned}\right\} \quad \ldots \quad (44\cdot7)$$

where

$$\left.\begin{aligned} A'_1 &= -\frac{c\kappa}{w}\left[A_1 + \tfrac{1}{2}(A_1 + A_2)\left(\frac{w^2}{\kappa^2 c_1^2} - 1\right)\right] \\ A_2' &= +\frac{c\kappa}{w}\left[A_2 + \tfrac{1}{2}(A_1 + A_2)\left(\frac{w^2}{\kappa^2 c_1^2} - 1\right)\right] \end{aligned}\right\} \quad (44\cdot8)$$

2. Japolsky [6] has taken these equations as meaning a system of rotating electromagnetic waves called "whirls" stretching from $-\infty$ to $+\infty$.

By assuming that they are real, and that they do not scatter away (as waves in a medium should) but somehow remain stable and even retain their shape, he has been able to deduce the most remarkable result

$$c_2.\, v_z = c^2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (44\cdot9)$$

where $c_2 = -\frac{w}{\kappa}$, the phase velocity, and v_z is the velocity with which the whole wave train with its central plane moves in the axial direction.

He has obtained de Broglie's formula in the form

$p.\lambda = \frac{h_1}{q}\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)$, which would make the product vary with v.

He also appears to have obtained certain other important results including Relativity relations in a world of whirls. His general Theory of Relativity appears to be different as it depends on the actual structure of the system of whirls to which it is applied. I have not had the opportunity to check these.

SECTION IV

The Rotational Theory of Light.

A SIMPLER PHYSICAL METHOD.

1. In the Rotational theory of light there is a complete picture of a binary system, with its two components of equal and opposite charges rotating round their common centre of gravity, which is moving forward along the helical axis of symmetry with a uniform velocity. When the two components, the poson and the negon, of the light radion have equal masses, they must describe the same circular orbit. In such a case there can be no question of the rotation of the orbit and the force components must be independent of θ itself. Also as the two opposite charges always remain at the ends of a diameter, the phase difference between the radial and the transverse components of the force is $\frac{\pi}{2}$. Their orbit may have any inclination to the sectional plane at right angles to the axis of symmetry and moving with the system i. e. it may not be perpendicular to the Z−axis.

In Ch. XIV. Sec V, it was as a first approximation tacitly assumed that the forces acted instantaneously, and that in a circular orbit the transverse component $Q = 0$. Corrections for the rotating charged components were postponed (see para 4 p. 355). Now it is *not* assumed that $E_\theta = 0$. Similarly we may consider that the value of E_z is not known to be zero, though it may turn out to be so.

We first consider the motion with respect to fixed axes and origin at the centre. As after a complete period of revolution, the same conditions revert at every point, it is obvious that the force components must be periodic in time t, and z, but independent of θ. Also as the forces must depend on the distance of the point, which is ordinarily very large compared to the diameter of the binary orbit, they must be functions of r. Hence we can put

$$\left.\begin{aligned} E_r &= R.\cos\phi \\ E_\theta &= \theta.\sin\phi \\ E_z &= Z.\cos(\phi+\gamma). \end{aligned}\right\} \quad \ldots \quad \ldots \quad (45{\cdot}1)$$

where $\phi = wt + \kappa z$, w and κ being some unknown constants and R, θ, Z are functions of r only and γ denotes the phase difference of Z.

2. Substituting these values in the equations (43·2) and putting $c_1 = \sqrt{\frac{c}{\mu k}}$, we get

$$\left.\begin{aligned} -\frac{w^2}{c_1^2}.R\cos\phi &= -\kappa^2.R.\cos\phi \\ &\quad + \kappa\frac{\partial Z}{\partial r}(\sin\phi.\cos\gamma + \cos\phi.\sin\gamma) \\ -\frac{w^2}{c_1}\theta.\sin\phi &= -\kappa^2.\theta.\sin\phi + \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial(r\theta)}{\partial r}\right]\sin\phi \\ -\frac{w^2}{c_1^2}.Z.(\cos\gamma.\cos\phi - \sin\gamma.\sin\phi) &= \frac{\kappa}{r}\frac{\partial}{\partial r}(r.R)\sin\phi \\ &\quad + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial Z}{\partial r}\right)\left[\cos\gamma.\cos\phi - \sin\gamma.\sin\phi\right] \\ \frac{1}{r}\frac{\partial(rR)}{\partial r}.\cos\phi - \kappa Z(\sin\gamma.\cos\phi &+ \cos\gamma.\sin\phi) = 0 \end{aligned}\right\} \ldots(45{\cdot}2)$$

Now all these equations hold for all values of ϕ; but $\sin\phi$ and $\cos\phi$ vary differently. Hence the co-efficients of $\sin\phi$ and $\cos\phi$ in each must when taken together separately vanish.

Therefore

$$\left(\frac{w^2}{c_1^2} - \kappa^2\right)R + \sin\gamma.\kappa\frac{\partial Z}{\partial r} = 0 \quad \ldots \quad \ldots \quad (45{\cdot}3)$$

$$\cos\gamma.\kappa\frac{\partial Z}{\partial r} = 0 \quad \ldots \quad \ldots \quad (45{\cdot}4)$$

$$\left(\frac{w^2}{c_1^2} - \kappa^2\right)\theta + \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial (r\theta)}{\partial r}\right] = 0 \quad \ldots \quad \ldots \quad (45{\cdot}5)$$

$$\left[\frac{w^2}{c_1^2} Z + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial Z}{\partial r}\right)\right]\cos\gamma = 0 \quad \ldots \quad \ldots \quad \ldots \quad (45{\cdot}6)$$

$$-\left[\frac{w^2}{c_1^2} Z + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial Z}{\partial r}\right)\right]\sin\gamma + \frac{\kappa}{r}\frac{\partial}{\partial r}(r.\,R) = 0 \ldots \quad (45{\cdot}7)$$

$$\frac{1}{r}\frac{\partial (r\,R)}{\partial r} - \sin\gamma .\,\kappa\, Z = 0 \quad \ldots \quad \ldots \quad \ldots \quad (45{\cdot}8)$$

$$-\cos\gamma .\,\kappa\, Z = 0 \quad \ldots \quad \ldots \quad \ldots \quad (45{\cdot}9)$$

As the orbit may have any phase difference $\cos\gamma$ is not necessarily zero. Also if the orbit is inclined forces must be some function of Z, hence $\kappa \neq 0$

It follows from (45·9) that $Z = 0$ ... ... (45·10)

This might have been foreseen from the symmetry.

This satisfies (45·4) also.

Then from (45·8) we have $\frac{1}{r}\frac{\partial (r\,R)}{\partial r} = 0.$

Hence $R = \frac{A}{r}$, where A is some constant (45·11)

From (45·3) we have $\left(\frac{w^2}{c_1} - \kappa^2\right) R = 0.$

As R is not always zero, $\frac{w^2}{c_1^2} - \kappa^2 = 0$ or $\frac{w}{\kappa} = \pm c_1$ ... (45·12)

Then from (45·5) $\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial (r\theta)}{\partial r}\right] = 0$

Hence $\theta = C.r + \frac{B}{r}$, where B and C are some constants (45·13)

Hence

$$\left.\begin{aligned} E_r &= \frac{A}{r}.\cos(wt + \kappa z) \\ E_\theta &= \left(Cr + \frac{B}{r}\right).\sin(wt + \kappa z) \\ E_z &= 0 \end{aligned}\right\} \quad \ldots \quad (45{\cdot}14)$$

It is obvious that E_θ must vanish at ∞. Therefore $C = 0$,

3. We can write these in the more usual forms

$$\left.\begin{aligned} E_r &= \frac{A}{r} \cdot \cos\frac{2\pi}{\lambda}(z-ut) \\ E_\theta &= \frac{B}{r} \cdot \sin\frac{2\pi}{\lambda}(z-ut) \\ E_z &= 0 \end{aligned}\right\} \quad \ldots \quad \ldots \quad (45{\cdot}15)$$

where λ represents the wavelength associated with the phase velocity u. The system is a plane wave moving along the Z-axis, with amplitude diminishing inversely as the distance from the axis increases.

4. In order to keep a constant phase, $wt + \kappa z = \text{constant}$. Hence z must vary as $-\frac{w}{\kappa} \cdot t$, and only differ by a constant. Hence the phase velocity, say u, of the system is $-\frac{w}{\kappa}$, which from (45·12) is equal to $\pm c_1$ according as the motion is upwards or downwards.

Thus the phase velocity is equal to the characteristic velocity of the medium,

$$u = c_1 = \frac{c}{\sqrt{\mu k}}. \quad \ldots \quad \ldots \quad \ldots \quad (45{\cdot}16)$$

It is also to be noted that no matter howsoever the orbit is inclined to the helical axis of symmetry i. e. the Z-axis there is no force component along this axis and there is an automatic propagation of the system along the axis of symmetry with the constant velocity $\frac{c}{\sqrt{\mu k}}$.

Accordingly the electromagnetic radionic system is identified with "light".

5. Even if R, θ and Z were not only functions of r but also of t, it is obvious from the fourth equation of (45.2) in which differentiation with respect to time does not occur that (45·8) and (45·9) still hold good. Hence Z remains zero.

6. It can be at once seen that

$$\left.\begin{aligned} \frac{\partial^2 E_r}{\partial t^2} &= \frac{w^2}{\kappa^2}\frac{\partial^2 E}{\partial z} \\ \text{and} \qquad \frac{\partial^2 E_\theta}{\partial t^2} &= \frac{w^2}{\kappa^2}\frac{\partial^2 E}{\partial z^2} \end{aligned}\right\} \quad \ldots \quad \ldots \quad (45{\cdot}17)$$

Thus the rotatory system represents an *automatic* wave propagation with the velocity $\pm\frac{w}{\kappa}$.

7. More generally, if the Z-component of the forces remains constant and they are also symmetrical round the Z-axis, then $\partial z = 0$ and

$$\frac{\partial}{\partial\theta} = 0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (45{\cdot}18)$$

Equations (43·2) then become

$$\left.\begin{aligned} \frac{1}{c_1^2}\frac{\partial^2 E_r}{\partial t^2} &= \frac{\partial^2 E_z}{\partial z^2} && \ldots \quad \ldots \quad (45{\cdot}19)\\ \frac{1}{c_1^2}\frac{\partial^2 E_\theta}{\partial t^2} &= \frac{\partial^2 E_\theta}{\partial z^2} + \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial (rE_\theta)}{\partial r}\right] && \ldots \quad \ldots \quad (45{\cdot}20)\\ 0 &= -\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial E_r}{\partial z}\right) && \ldots \quad \ldots \quad (45{\cdot}21)\\ \frac{1}{r}\frac{\partial}{\partial r}(r\,E_r) &= 0 && \ldots \quad \ldots \quad (45{\cdot}22) \end{aligned}\right\}$$

From (45·19) E_r is an automatic wave propagation with velocity c_1.

From (45·22) $E_r = \frac{A}{r} f_r\,(t, z, \theta)$, which satifies (45·21).

From (45·20) it is clear that E_θ also will be an automatic wave propagation with the same velocity c_1 if $\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial (rE_\theta)}{\partial r}\right] = 0$

which gives $$E_\theta = \left(\frac{B}{r} + Cr\right) f_\theta(t, z, \theta).$$

And $$E_z = \text{constant}.$$

8. Equations (43·2) will hold not only for the region free from the charges, but also at the two infinitesimal points of the charges if $\frac{\partial}{\partial t}(\rho v) = 0$. In the region free from charges $\rho = 0$. At the charges v should be constant. If the motion is uniform and circular, v and its components v_r, v_θ, v_z will remain constant, and the equations will be satisfied.

SECTION V

The Rotational Theory of Matter.

1. As has been actually adopted by N. S. Japolsky, ϕ can be a complex function, for it is not absolutely necessary to take the simple case when all the coefficients are real. They can be expressed in a complex form and yet their combined result may remain real.

Consider $i\,\phi = i\,(\phi_1 + i\,\phi_2) = i\,(w_1 t + \kappa_1 z + n_1\theta) - (w_2 t + \kappa_2 z + n_2\theta)$

From (44·5) it is clear that $\sqrt{\frac{w^2}{c_1^2} - \kappa^2}$ is real.

Hence $\sqrt{\frac{w_1^2 - w_2^2}{c_1^2} - \kappa_1^2 + \kappa_2^2 + 2i\left(\frac{w_1 w_2}{c_1^2} - \kappa_1\kappa_2\right)}$ is real

Therefore $\frac{w_1 w_2}{c_1^2} = \kappa_1 \kappa_2$ ... ... (46·1)

But $-\frac{w_1}{\kappa_1}$ represents the real phase velocity u, because in order to maintain the phase constant, z must increase with the velocity so as to keep $w_1 t + \kappa_1 z = 0$

Hence $u = -\frac{w_1}{\kappa_1}$ ... ... (46·2)

2. The solutions of Maxwell's equations hold for z referred to the origin O and the axes O_x, O_y, O_z. If axes be taken in the system moving with it relatively to O along the Z-axis, and v be the velocity of such relative motion.

then $z = \xi + vt$

Substituting we get

$$\left.\begin{aligned}\phi_1 &= w_1 t + \kappa_1 (\xi + vt) + n_1\theta \\ \phi_2 &= w_2 t + \kappa_2 (\xi + vt) + n_2\theta\end{aligned}\right\} \quad \ldots \quad \ldots \quad (46·3)$$

Now if ϕ_2 were to vary with time, the function will cease to be periodic. But the moving binary system is perfectly stable.

Hence $w_2 + \kappa_2 v = 0$

$\therefore$ $v = -\frac{w_2}{\kappa_2}$ ... ... (46·4)

From (46·1), (46·2) and (46·4) we get $u.v = c_1^2$ ... ... (46·5)

This can be identified with the well-known DE BROGLIE'S formula that the product of the velocity of the associated De Broglie wave and the velocity of the particle is equal to the square of the velocity of light in a vacuum. For a medium where $\mu \neq k \neq 1$ the product equals the square of the velocity in the medium.

Hence the phase velocity u is identified with that of the mysterious, and obscure De Broglie waves.

3. In the case of Light. as $u = c_1$ it follows that $u = v = c_1$

... ... (46·6)

Momentum × Wavelength $= p.\lambda = mc_1 . c_1 T = mc_1^2 . T$

$$= E.T = \frac{E}{\nu} = h \quad \ldots \quad \ldots \quad (46{\cdot}7)$$

SECTION VI

Relativity Postulates—A Misinterpretation.

We know from (46·5) that $u.v = c_1^2$, and from (46·2) $u = -\frac{W_1}{K_1}$. We may put $\kappa_1 = \frac{2\pi}{\lambda}$

Accordingly we can write

$$\phi_1 = \frac{2\pi}{\lambda}(z - ut) + n\theta$$

$$= \frac{2\pi}{\lambda}\left(z - \frac{c_1^2}{v}t\right) + n\theta \quad \ldots \quad \ldots \quad (47{\cdot}1)$$

Also from (46·3)

$$\phi_1 = \kappa_1\left[\xi - (u - v)t\right] + n\theta$$

$$= \frac{2\pi}{\lambda}\left[\xi - \left(\frac{c_1^2}{v} - v\right)t\right] + n\theta$$

$$= \frac{2\pi}{\lambda}\left[\xi - \frac{c_1^2}{v}\left(1 - \frac{v^2}{c_1^2}\right)t\right] + n\theta$$

$$= 2\pi\left(\frac{\sqrt{1 - \frac{v^2}{c_1^2}}}{\lambda}\right)\left[\frac{\xi}{\sqrt{1 - \frac{v^2}{c_1^2}}} - \frac{c_1^2}{v}\left(\sqrt{1 - \frac{v^2}{c_1^2}}.t\right)\right]$$

... ... (47·2)

Comparing (47·1) and (47·2) it is seen at once that the quantity ϕ_1 will retain exactly the same form and can be assumed to be an invariant whether (1) we express it with reference to fixed axes for which z stands, or (2) express it with reference to the axes moving relatively to them, for which stands ξ, *provided* in the latter case we alter the measuring units of Time and Space in such a way as to treat the new units as

$$\sqrt{1-\frac{v^2}{c_1^2}}\,.\,t' = t. \text{ and } \sqrt{1-\frac{v^2}{c_1^2}} = l \qquad \ldots \quad (47\cdot3)$$

These are the famous Lorentz transformations. It is obvious that with these altered units Maxwell's equations remain unchanged. This circumstance can be *wrongly* interpreted as if for a system moving with a velocity, the time expands and the length contracts. With due deference to the conclusion of Japolsky, *the real truth is that if a system is both rotating and moving forward with a uniform velocity, then vector quantities, which are subject to Maxwell's equations, do change, but can be wrongly assumed to remain an invariant if Lorentz transformations were applied. But this would not hold in the case of a simple translatory motion.*

It was shown in Chapter XIII Sec. II that the Space-Time continuum of the General Theory of Relativity is a partial misinterpretation of a spherical wave propagation. It is now clear that the postulates of Special Relativity are a partial misinterpretation of the effect of a rotatory cum translatory motion.

SECTION VII

Quantum or Wave Mechanics—A mere Rotational Mechanics.

1. We may consider any binary system, e. g. a hydrogen atom, with its two components of equal and opposite charges, rotating round each other, with their common centre of gravity moving forward with a uniform velocity v. In such a system the period of a complete revolution T can remain constant, while v can be different on ejection.

The wave length associated with the velocity of the centre of gravity given by $\lambda = vT$ is in a submicroscopic system incapable of

measurement. But the wave-length associated with the phase velocity $\lambda = u\,T$ can be measured, where T is a constant.

Now $$u.v = c_1^2 = \frac{c^2}{\mu k}$$

Let the linear momentum of the whole system along the Z axis be p and the total mass m.

Then $$p.\lambda = mv.uT = \frac{mc^2}{\mu k}.\ T = \frac{mc^2}{\mu k}.\ \frac{1}{\nu} = h \quad \ldots \quad \ldots \quad (48{\cdot}1)$$

This is the well-known relation assumed by DE BROGLIE, which is here not assumed, but actually deduced.

In the case of light, mc^2 equals the energy (see Ch. XIV, para. 5 p. 360). If. with EINSTEIN. we put the constant here also as E, the energy,

then $$E = mc^2 \quad \ldots \quad \ldots \quad \ldots \quad (48{\cdot}2)$$

PLANCK'S assumption $E = h\nu$ then follows automatically. $\ldots$ (48·3)

It is quite wrong to confuse energy with mass. In the electromagnetic free rotatory binary system, which is also moving forward, the kinetic energy is the sum total of their kinetic energies obtained from their translatory as well as rotatory motions. In Newton's Mechanics kinetic energy $= \frac{1}{2} mv^2$, which varies with v. In the rotatory system the forward velocities of the two components are so related with their circumferential velocities that the combined kinetic energy remains constant.

As r and θ are not separately measured, and the value repeats itself after each revolution we can at a long distance take the average value of

$$f(r)\, e^{i n \theta} = A \text{ as constant.}$$

Hence $$E = A\ e^{2\pi i (z - ut)} \quad \ldots \quad \ldots \quad \ldots \quad (48{\cdot}4)$$

If the axis of symmetry, along which the system is propagated, be not taken as the z-axis, but any direction r. with direction cosines (l', m', n') then the equation when referred to new fixed Cartesian coordinates becomes

$$\psi = A\, e^{2\pi i \left(\frac{mv}{h}\right)\left\{l'x + m'y + n'z - \frac{c_1^2}{v} t\right\}} \quad \ldots \quad (48{\cdot}4)$$

where ψ has been substituted for E, because the same result holds for any vector quantity of the same type, which is subject to Maxwell's equations.

3. We now get from (49·4)

$$\left.\begin{aligned} \frac{\partial\psi}{\partial x} &= \frac{2\pi i}{\lambda}\, l'\psi \quad ; & \frac{\partial^2\psi}{\partial x^2} &= \left(\frac{2\pi i}{\lambda}\right)^2 l'^2.\ \psi \\ \frac{\partial\psi}{\partial y} &= \frac{2\pi i}{\lambda}\, m'\psi \ ; & \frac{\partial^2\psi}{\partial y^2} &= \left(\frac{2\pi i}{\lambda}\right)^2 m'^2\,\psi \\ \frac{\partial\psi}{\partial z} &= \frac{2\pi i}{\lambda}\, n'\psi \ ; & \frac{\partial^2\psi}{\partial z} &= \left(\frac{2\pi i}{\lambda}\right)^2 n'^2\,\psi \\ \frac{\partial\psi}{\partial t} &= -\frac{2\pi i}{\lambda}\, u\,\psi \ ; & \frac{\partial^2\psi}{\partial t^2} &= \left(\frac{2\pi i}{\lambda}\right)^2 .\ u^2\,\psi \end{aligned}\right\} \quad \ldots\ \ldots \quad (48\cdot5)$$

$$\text{Hence} \quad \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} - \frac{1}{u^2}\,\frac{\partial^2\psi}{\partial t^2} = 0\ , \text{ as } l'^2 + m'^2 + n'^2 = 1 \quad \ldots \quad \ldots \quad \ldots \quad (48\cdot6)$$

which is a wave propagation with velocity u.

Also

$$\left.\begin{aligned} \frac{\partial\psi}{\partial x} &= \frac{2\pi i m}{h}\cdot v_x\,\psi & \frac{\partial}{2\pi i}\,\frac{\partial}{\partial x} &= m\,v_x = p_x \\ \frac{\partial\psi}{\partial y} &= \frac{2\pi i m}{h}\, v_y\,\psi & \frac{h}{2\pi i}\,\frac{\partial}{dy} &= m\,v_y = p_y \\ \frac{\partial\psi}{\partial z} &= \frac{2\pi i m}{h}\, v_z\,\psi & \frac{h}{2\pi i}\,\frac{\partial}{\partial z} &= m\,v_z = p_z \\ \frac{\partial\psi}{\partial t} &= -\frac{2\pi i m}{h}\, c_1^2\,\psi & \frac{h}{2\pi i}\,\frac{\partial}{\partial t} &= m\,c_1^2 = E \end{aligned}\right\} \quad \ldots \quad (48\cdot7)$$

It should however be remembered that these are *purely artificial devices* for representing momenta and energy, and do not in fact represent reality.

4. From (49·5) we also have

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \left(\frac{2\pi i}{\lambda}\right)^2 .\ \psi\,(l'^2 + m'^2 + n'^2) = \left(\frac{2\pi i}{\lambda}\right)^2 \psi$$

$$\text{Hence} \quad \nabla^2\psi + \left(\frac{2\pi}{\lambda}\right)^2 \psi = 0 \quad \ldots \quad \ldots \quad (48\cdot8)$$

This is identified with the famous wave equation of HEISENBERG and SCHRODINGER.

SECTION VIII

The Moving Atom

1. As the period of rotation is T, the rotation of each component in the projection on the x y plane may be taken to be in the ellipse (a, b) so that

$$x = a\cos\left(\frac{2\pi}{T}t+\alpha\right) \text{ and } y = b\cos\left(\frac{2\pi}{T}t+\beta\right)$$

Hence

$$\frac{dx}{dt} = -\frac{2\pi a}{T}\sin\left(\frac{2\pi}{T}t+\alpha\right) \text{ and } \frac{dy}{dt} = -\frac{2\pi b}{T}\sin\left(\frac{2\pi}{T}t+\beta\right)$$

Therefore the mean energy of the two during the complete period is

$$W = \frac{1}{T}\int_0^T \frac{m}{2}\left[\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2\right]dt = \frac{m\pi^2}{T^2}(a^2+b^2). \quad \ldots \quad (49\cdot1)$$

2. If the components of v be v_x, v_y and v_z, then

$$v_x = l'v = l'.\frac{h}{m\lambda} \; ; \; v_y = m'.\frac{h}{m\lambda} \; ; \text{ and } v_z = n'.\frac{h}{m\lambda}.$$

Now if $\bar{E}$, V and K stand respectively for the total energy of the system, the potential energy of the electron, and the kinetic energy of the translatory motion of the whole system, and $\bar{E}' = \bar{E} - W$, then from the principle of the conservation of evergy

$$E' - V = K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m(v_x^2+v_y^2+v_z^2).$$

$$= \tfrac{1}{2}\left(\frac{h}{m\lambda}\right)^2, \text{ where m is the total mass of the atom.}$$

Hence

$$\frac{8\pi^2 m}{h}(\bar{E}'-V) = \left(\frac{2\pi}{\lambda}\right)^2 \quad \ldots \quad \ldots \quad (49\cdot2)$$

Accordingly from (48·8) we get

$$\nabla^2\psi + \frac{8\pi^2 m}{h^2}(\bar{E}'-V)\psi = 0$$

or

$$\left[\nabla^2+\frac{8\pi^2 m}{h^2}(\bar{E}'-V)\right]\psi = 0 \quad \ldots \quad \ldots \quad (49\cdot3)$$

This is similar in form to the famous HEISENBERG-SCHRODINGER equation. As for a freely moving atom $(E-V)$ is constant, this is the equation of a plane wave. Accordingly atoms will show diffraction as has already been observed by *Dempster*.

3. The falsity of the artificial devices becomes exposed when the interaction of n atoms is considered. Heisenberg-Schrodinger's equation then becomes

$$\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial y_1^2}+\frac{\partial^2}{\partial z_1^2}\right)+\left(\frac{\partial^2}{\partial x_2^2}+\frac{\partial^2}{\partial y_2^2}+\frac{\partial^2}{\partial z_2^2}\right)+\ldots$$

$$+\left(\frac{\partial^2}{\partial x_n^2}+\frac{\partial^2}{\partial y_n^2}+\frac{\partial^2}{\partial z_n^2}\right)+\frac{8\pi^2 m}{h^2}(E-V)\Big]\psi=0$$

which represents an imaginary wave propagation in a space of $3n$ dimensions.

4. The Rotational Theory steers clear of such an impossible mathematical fiction. If any number of rotating systems have the same wave length and the same phase velocity, then they can be easily compounded.

$$E_r=\sum_1^n E_{nr}=\sum_1^n\left\{R_n\,.\,e^{\frac{2\pi i}{\lambda}(z-ut+\alpha_n)}\right\}$$

$$=R\,.\,e^{\frac{2\pi i}{\lambda}(z-ut+\alpha)} \qquad \ldots \quad \ldots \quad \ldots \quad (49{\cdot}4)$$

where $$R=\sqrt{(\Sigma R_n\cos\alpha_n)^2+(\Sigma R_n\sin\alpha_n)^2}$$

and $$\tan\alpha=\frac{\sum_1^n R_n\sin\alpha_n}{\sum_1^n R_n\cos\alpha_n}.$$

We get similar values for E_θ and E_z.

SECTION IX

Rotating Magnet.

1. A glance at the equations (43·2) and (43·3) would show that they are exactly identical. Hence if we take

$$M = f(r)\, e^{i(wt + kz + n\theta)}$$

we obtain for the magnetic intensity exactly the same form as in the preceding Sections. Hence a rotating magnetic diapole will produce a similar field as a rotating electric diapole.

2. Thus if we have a rotating magnet, with its positive and negative poles we get the same result. If the distance between the poles is variable they must be connected by some sort of an elastic band. But if the distance between them is not variable, they would each describe circles round the common centre of gravity, even if the latter is not equidistant from the two poles. In such a case there would be no question of any rotation of the orbits, and the results would be identical with those in Sec. IV with $n = 0$.

Thus a light radion (though this is not probable) can just as well be a rotating magnet, instead of a binary system with its two component charges separated.

3. It will be seen that not only the whole system, but each component is subject to the equations (43.2). Hence all the results of Sec. IV apply to a single electron moving round a proton, because the force acting on it is always radial. A glance at the figure in Ch. XIV, Sec. VI, P. 358 will show that even a single rotating electron can emit an electromagnetic wave.

The effect of a spinning magnet, behaving in a similar way, will be considered later in connection with the diffraction of free electrons for which the potential energy $= 0$.

APPENDICES

A. Appendix to Ch. II. Sec VI para 3.

MICHELSON AND MORLEY'S EXPERIMENT

It was pointed out there that for this experiment monochromatic light produced on the Earth has been used so far, and that like ordinary material particles it may already possess the velocity of the earth in addition to its own.

Hence in the experiment $\frac{1}{(c-v)+v} + \frac{1}{(c+v)-v} \equiv \frac{1}{c} + \frac{1}{c}$

It follow that any experiment in which *terrestrial* light is used is inconclusive and does not therefore prove the postulate of Relativity. To have a crucial test, it is necessary to use light coming from outside the earth, so that its velocity would be independent of that of the Earth.

Although in view of (47·3) and the physical explanation suggested in Ch. IV. Sec. II para. 2 p. 230, it is just possible that the velocity of light may be wholly independent of that of its source yet that is not likely.

AN EXPERIMENT SUGGESTED

Although for proving interference, it is not absolutely necessary to have monochromatic light, such light can conveniently be obtained, which is not produced on the Earth.

Light from the Sun should first be passed through a series of prisms (as in a monochromator), then all other light, except that with the chosen wavelength, intercepted by obstacles, and only nearly monochromatic light allowed to fall on a plane reflector and then directed into the Michelson-Morley Apparatus. Although the intensity will be diminished, there will be a greater facility to observe the interference fringes.

AN APPEAL

I earnestly appeal to experimenters to repeat the experiment with Solar light which will test whether light is corpuscular in character and also whether the postulate of Relativity is not sound. I venture to make the PREDICTION that *there will no longer be the null effect.*

B. Appendix to Ch. XIII Sec. II p. 278

ADDENDUM

In May last an Addendum to Ch. XIII Sec. II p. 278 was printed and circulated, pointing out that although P, Q and R must be symmetrical functions of x,y and z, it had been unnecessarily thought that i Ddt also must come in similarly. On reconsideration it seemed that this may not be so. The reason is that although the two equations of the gravitational wave are symmetrial with regard to x, y, z, and w, the orbital equation of the particle, unaffected by gravitation is not so because of v.

Without any such restriction as to symmetry, if the gravitational influence is superimposed, the orbital equation would be

$$(1+f_1)\,dr^2 + (1+f_2)\,r^2\,d\theta^2 + (f_3 - v^2)\,dt^2 = 0 \quad \ldots \quad \ldots \quad (50{\cdot}1)$$

where the f's are some unknown functions. From symmetry round the origin they must be independent of θ; and as the influence would be proportional to the gravitating mass M at the origin, they must contain M as a factor; remembering that they must vanish at ∞ and expanding them by Laurent's theorem we get

$$(1 + M\sum_1^\infty \frac{A_n}{r^n})\,dr^2 + (1 + M\sum_1^\infty \frac{B_n}{r^n})\,r^2\,d\theta^2 + (M\sum_1^\infty \frac{C_n}{r^n} - v^2)\,dt^2 = 0.$$

where, the A's, B's and C's are constants and are known to be small. Snbstituting $r^2\frac{d\theta}{dt} = h$, and retaining terms of the order $\frac{1}{r^3}$ only we get

$$\left(\frac{dr}{r^2 d\theta}\right)^2 + (1+\frac{MB_1}{r})\,\frac{1}{r^2} = \frac{v^2}{h^2} - \frac{MC_1}{h^2}\cdot\frac{1}{r}$$

or

$$\left(\frac{du}{d\theta}\right)^2 + (1+MB_1.\,u)\,u^2 = \frac{v^2}{h^2} - \frac{MC_1}{h^2}\cdot u$$

Hence

$$\frac{d^2u}{d\theta^2} + u = -\frac{MC_1}{2h^2} - \frac{3MB_1}{2}\cdot u^2$$

$$= \frac{GM}{h^2} + \frac{3GM}{D^2}\cdot u^2 \quad \ldots \quad (50{\cdot}2)$$

where $G = -\frac{C_1}{2}$ is identified with the gravitational constant, and $D = +\frac{C_1}{B_1}$ is another universal constant found to be equal to the velocity of light.

ADDENDUM

to Sec. IV, para. 2, page 74.

Even without the particular case that $C = 0$, the solution

$$E_\theta = (Cr + \frac{B}{r}) \sin(\omega t + kz)$$

can admit of a physical explanation.

As r increases, E_θ does not approach infinity, but a limit is reached when the attractive force is so much diminished that the components part company and proceed on parabolic paths. The binary system therefore merely ceases to exist beyond this limit.

Again, as r diminishes, a limit is reached when the force of attraction is so great that the two components rush towards each other and coalesce, and the binary system ceases to exist. Inside this limit the binary rotating system cannot be preserved without an extremely large transverse force ; and as such large force does not exist the binary system itself breaks down.

Thus we have physical explanations of Sir J. J. Thomson's otherwise inexplicable "two cores". (*Nature* Vol. 137 pp. 232-3)

References

1. Eddington : *Nature of the Physical World*, p. 179.
2. Einstein : *The World As I See It*, p. 138.
3. Padolsky : *Phil. Mag. Vol.* XXII, p. 998.
4. Sulaiman : *Ind. Phy-Math. Journ.* VIII p. 12.
5. Japolsky : *Phil. Mag. Vol.* XIX pp. 93-4.
6. Do. : *Ibid. Vol* XX pp. 417-468.

ON A SIMPLE DERIVATION OF STRESSES IN A MOVING FLUID

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Received November 26, 1936

SUMMARY

This note describes a simple method of deriving equations for internal forces in a fluid which is in relative motion, paying special attention to the physical principles involved.

Stokes[1] derived the equations for stresses in a fluid which was in motion by considerations of internal friction and showed that the pressures which remain the same in all directions in the case of motionless fluid, vary in different directions when internal friction is present. His method of derivation though lengthy is very instructive. While Lamb[2] and others have obtained the same formulæ by a process which does not throw any light into the internal mechanism of the shifting motion and the friction that results from such a type of fluid motion. Take for instance a plane sound wave travelling along x axis, the pressure p_1 in the same direction is given by

$$p_1 = -p + \tfrac{2}{3}\mu\Delta + 2\mu\dot{e}_1 \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

where p represents the hydrodynamical pressure, Δ the rate of volume dilatation and $\dot{e}_1$ the time rate of extension of fluid elements along x axis. In the present case

$$\dot{\Delta} = \dot{e}_1$$

and $$\dot{e}_1 = \frac{du}{dx},$$ where u represents the fluid velocity along x axis.

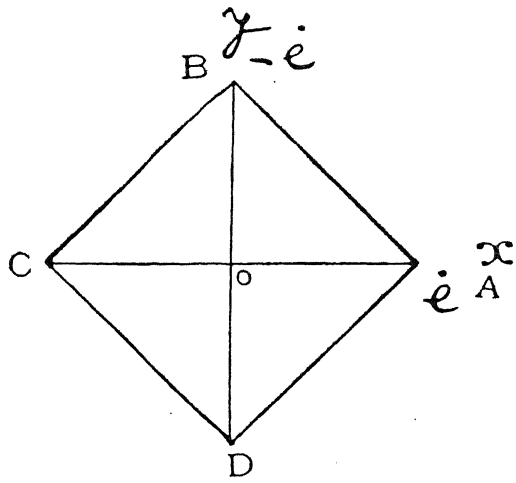
According to the definition of viscosity the tangential friction between two fluid layers in motion is given by

$$T = \mu\dot{\theta}$$

and $$\dot{\theta} = \frac{du}{dy} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

where $\dot{\theta}$ is the rate of shearing strain.

Thus from (2) we find that tangential friction arises when there is a gradient of u along y axis ; while (1) shows that the internal friction has entered into the formula when there is no variation of u along y axis, the wave being plane $\frac{du}{dy}=0$. Lamb's general formulæ could not throw any light on this point. And it was this difficulty that led the writer to investigate and derive the general formulæ from the fundamental definition i. e., equations (2) with the help of the principle of superposition of shifting motions laid down by Stokes.



Let us suppose that a cube of fiuid is isolated from the rest so that A B C D represents its section when it is at rest. Further let the motion of the fluid element take place in such a manner that extensions of elements paralle to O A and OB are parallel to x,y axes respectively, while there is no motion of fluid elements parallel to OZ, ie Z axis. Hence we can consider the deformation of the two dimensional square A B C D and find out the stresses that arise during motion. If the time rate of extensions of elements along OA (x axis) and O B (y axis) be $\dot{e}$ and $\dot{-e}$ then we know that these give rise to shifting motion along AB and we have

$$\theta = 2\dot{e} \quad T = 2\mu e \qquad \ldots \qquad \ldots \quad (3)$$

and while the normal stress across planes perpendicular to OA and OB along x and y axes are P and $-P$, and

$$P=T \qquad \ldots \qquad \ldots \qquad \ldots \quad (4)$$

If we put $P/e = a$ where a is at present an unknown constant, then from (3) and (4) we find $a = 2\mu$

thus
$$\left.\begin{aligned} P_1 &= 2\mu\, e \text{ along x axis} \\ P_2 &= -2\mu\, e \text{ along y axis} \end{aligned}\right\} \ldots \ldots \ldots \quad (5)$$

In the case when there are shifting motions along other directions also, the relation between the time rate of extensions and stresses can be obtained by the principle of superposition. Let us consider three cases represented in Figures (2) (3) and (4) which represent sections of the elemental cube of fluid in xy, xz, and zy planes respectively.

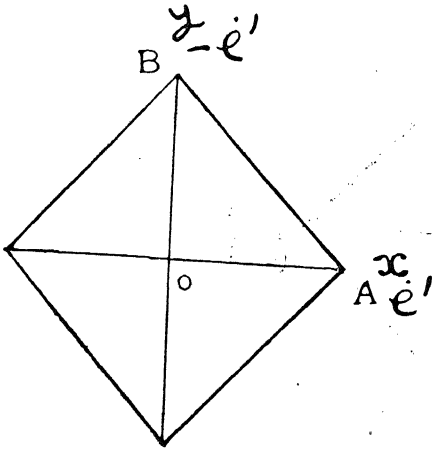


Fig. (2)

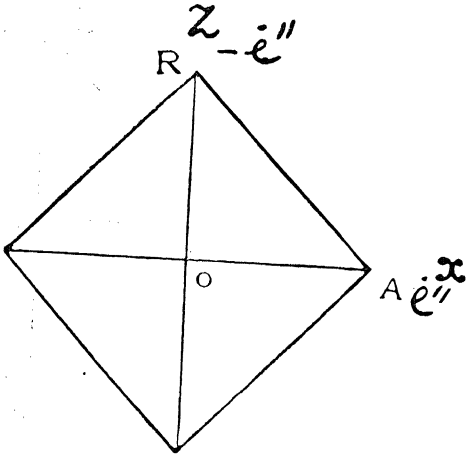


Fig. (3)

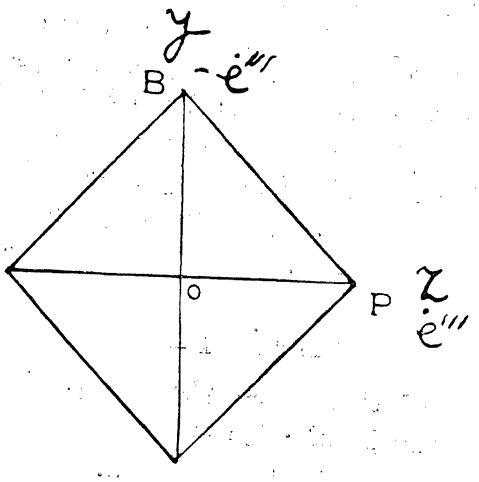


Fig. (4)

Let $\dot{e}'$ and $-\dot{e}'$ represent the rate of extensions of fluid elements along x and y axes for shifting motion parallel to A B Fig (2) so that all planes parallel to A B and perpendicular to the plane of the paper shift relatively to A B and give rise to a rate of shear of amount

$$\dot{\theta}_1 = 2\dot{e}'$$

Similarly let extensions alongOX and OZ Fig. (3) be $\dot{e}''$ and $-\dot{e}''$ which give rise to shifting motion parallel to AR; while Fig (4) represents extensions along OZ, and OY of magnitudes $\dot{e}'''$ and $-\dot{e}'''$ produce shifting motion along P B. These together represent the most general type of shifting motion that might occur in a moving fluid.

If X_x, Y_y and Z_z represent normal stresses along x, y, and z axes respectively across planes for which x, y and z have constant values,

then $X_x = 2\mu(\dot{e}' + \dot{e}'')$, $Y_y = 2\mu(-\dot{e}'-\dot{e}''')$,

and $Z_z = 2\mu(\dot{e}'''-\dot{e}'')$ by (5) ... ... (6)

And $\dot{e}_x = (\dot{e}'+\dot{e}'')$, $\dot{e}_y = -(\dot{e}'+\dot{e}''')$, and $\dot{e}_z = (\dot{e}''' - \dot{e}'')$ ... ... (7)

where $\dot{e}_x$, $\dot{e}_y$ and $\dot{e}_z$ represent the total extensions along x, y and z axes. Further we observe that

$$\dot{e}_x + \dot{e}_y + \dot{e}_z = 0$$

as should be the case in the case of pure shifting motions, hence for pure shifting motions the rate of dilatation $\dot{\delta} = 0$

Equations (6) can be written down as

$$Xx = 2\mu\dot{e}_x \quad Y_y = 2\mu\dot{e}_y \quad Z_z = 2\mu\dot{e}_z \quad \ldots \quad \ldots \quad (9)$$

If $\dot{e}x = \dot{e}y = \dot{e}z$ we have a rate of uniform dilatation. It will be shown presently that such a rate of dilatation does not produce any stress which depends upon the internal friction of fluid, that is, μ. From (7) we find that when $\dot{e}_x = \dot{e}_y = \dot{e}_z$

$$\dot{e}' = -\dot{e}'', \text{ and } \dot{e}' = -\dot{e}'''$$

and $X_x = Y_y = Z_z = 0$ ... ... (10)

It is therefore evident that from the general type of rates of extensions

$\dot{e}_x$, $\dot{e}_y$ and $\dot{e}_z$, the portion which is due to rate of uniform dilatation should be subtracted. Let us assume that $\dot{e}_x = \dot{e}'_x + \dot{e}$, $\dot{e}_y = \dot{e}_y' + \dot{e}$, $\dot{e}_z = \dot{e}_z' + \dot{e}$ where $\dot{e}'x$, $\dot{e}y'$ and $\dot{e}z'$ represent the rate of extensions for pure shifting motion and $\dot{e}$ the extension for uniform dilatation.

Putting $\dot{e}_x + \dot{e}_y + \dot{e}_z = \dot{\triangle}$ the dilatation, we observe that

$$\dot{\triangle} = \dot{e}_x' + \dot{e}_y' + \dot{e}_z' + 3\dot{e} = \dot{\delta} + 3\dot{e}$$

But by (8) $\dot{\delta} = 0$, hence $\dot{e} = \dot{\triangle}/3$

and
$$\left.\begin{aligned} X_x &= 2\mu(\dot{e}_x - \dot{\triangle}/3) \\ Y_y &= 2\mu(\dot{e}_y - \dot{\triangle}/3) \\ Z_z &= 2\mu(\dot{e}_z - \dot{\triangle}/3) \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

Now we are in a position to write down the equations for stresses when the hydrodynamical pressure p is also present. This pressure is the same in all direction, hence considering the forces on the face of a cube of fluid, we find

$$P_1 = -p + X_x$$

$$P_2 = -p + Y_y$$

$$P_3 = -p + Z_z$$

where P_1, P_2 and P_3 represent the total stress along x, y and z axes respectively. Substituting the values of X_x, Y_y and Z_z from (11)we get

$$\left.\begin{aligned} P_1 &= -p - \tfrac{2}{3}\mu\dot{\triangle} + 2\mu\dot{e}_x \\ P_2 &= -p - \tfrac{2}{3}\mu\dot{\triangle} + 2\mu\dot{e}_y \\ P_3 &= -p - \tfrac{2}{3}\mu\dot{\triangle} + 2\mu\dot{e}_z \end{aligned}\right\} \quad \ldots \quad \ldots \quad (1)$$

These are the fundamental equations for stresses in a fluid which is in and when viscosity is taken into consideration. The mean stress, motion $(P_1 + P_2 + P_3)/3 = -p$, the same as the hydrodynamical pressure.

We shall now illustrate our formulæ by a discussion of the case referred to in the introductory paragraphs.

Since there is no motion along y and z axes, $\dot{e}_y = \dot{e}_z = 0$ and $\dot{e}_x = \dot{e}_1$; $\dot{\triangle} = \dot{e}_1$, while $\dot{e} = \dot{e}_1/3$ by (11). Thus

$$P_1 = -p - \tfrac{2}{3}\mu\dot{e}_1 + 2\mu\dot{e}_1$$

$$P_2 = -p - \tfrac{2}{3}\mu\dot{e}_1$$

$$P_3 = -p - \tfrac{2}{3}\mu\dot{e}_1$$

And referring to equations (7) and (11) we find in the present case

$$\dot{e}_x' = \dot{e}_x - \frac{\dot{\triangle}}{3} = \dot{e}_1 - \frac{\dot{e}_1}{3} = \tfrac{2}{3}\dot{e}_1 = (\dot{e}' + \dot{e}'')$$

Similarly $\quad -\dot{e}' - \dot{e}''' = -\dot{e}_1/3$

and $\quad \dot{e}''' - \dot{e}'' = \dot{e}_1/3.$

From these relations we evaluate $\dot{e}'$, $\dot{e}''$ and $\dot{e}'''$. It is found that

$$\dot{e}''' = 0\ ,\ \dot{e}' = \dot{e}_1/3, \text{ and } \dot{e}'' = \dot{e}_1/3, \text{ and } \dot{e} = \dot{e}_1/3$$

Thus we find that this kind of motion is equivalent to (1) a rate of uniform dilatation of magnitude $\dot{\triangle} = \dot{e}_1$, and the shifting motions produced by extensions $\dot{e}' = \dot{e}_1/3$ along x axis, with an equivalent contraction along y axis; and another pair of extensions of magnitude $\dot{e}'' = \dot{e}_1/3$ along x axis, and $-\dot{e}_1/3$ along z axis. (Refer to Figs. 2 and 3). These two shifting motions with a rate of uniform dilatation produce the requisite motion $\dot{e}_1$ along x axis only, for

$$\dot{e}_x = \dot{e}'_x + \dot{\triangle}/3 = \dot{e}' + \dot{e}'' + \dot{e}_1/3$$

$$= \frac{\dot{e}_1}{3} + \frac{\dot{e}_1}{3} + \frac{\dot{e}_1}{3} = \dot{e}_1$$

$$\dot{e}_y = -\frac{\dot{e}_1}{3} + \frac{\dot{e}_1}{3} = 0$$

$$\dot{e}_z = -\dot{e}'' + \dot{\triangle}/3$$

$$= -\frac{\dot{e}_1}{3} + \frac{\dot{e}_1}{3}$$

$$= 0$$

Thus all the conditions of problem are satisfied.

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REFLECTION OF RADIO WAVES FROM THE IONOSPHERE

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(Communicated by Prof. M. N. Saha)

Received January 30, 1937

SUMMARY

Assuming that considerable reflection of radio waves takes place in the ionosphere at a place where electronic density is such that $\frac{d\mu}{dN}$ is very large, it has been shown that the ordinary wave can be reflected from only one concentration of electrons while the extraordinary wave has in general three conditions of reflection for values of $p > p_H$.

Our experimental result that the penetration frequency difference between the extraordinary and the ordinary rays has on several occasions been observed to be only ·14mc/sec. when the frequency of the incident waves was about 4 mc/sec has been explained in the light of the theoretical results obtained.

It has been shown that reflection of radio waves from the ionosphere can take place not only from the places where group velocity is zero and phase velocity infinite but also from the strata where both the velocities are together zero.

Introduction

Since the year 1901, the problem of the propagation of radio waves has been attracting much attention and within the last few years a large amount of work has been done on the propagation of these waves through the upper ionized regions under the influence of the earth's magnetic field. The magneto-ionic theory developed by Breit, Appleton, Goldstein and Hartree has many notable successes to its credit. Appleton [1] generalized the treatment given by Lorentz and obtained the general expressions for dispersion and polarization of the waves. The same result was obtained by Hartree [4] in an altogether different manner. Martyn [5] has drawn dispersion and polarization curves for a number of wave lengths taking different values of collisional frequency and three values of θ (0°, 45° and 90°) the angle between the direction of the earth's magnetic field and

the direction of propagation of the waves. Mary Taylor [8,9] has also drawn the same curves with and without friction though only for Slough (England). All these investigators have assumed that the reflection of waves takes place from the place where refractive index becomes zero. Booker[2], however, in his theoretical exposition of the various important aspects of the problem has mentioned that appreciable reflection of waves will take place when either the complex refractive index q becomes approximately equal to zero, or the change of q per vacuum wave length is considerable. He has neglected the latter condition remarking that it would be applicable to long waves and the whole of his treatment is based on the former condition only. The following analysis will show that the second condition applies not only to long waves but to short waves as well and that it is in fact the general condition which contains the former condition as a special case.

Theoretical

We know that the refractive index changes its value with the variation in the electron density N. It is also well known that in the ionosphere there exists an ionization gradient, i.e. the ionization density increases with height. Thus the condition that 'when change in the refractive index per vacuum wave length is considerable', may very well be taken to be the same as 'when change in refractive index for small changes in the value of N, the ionization density, is large', or to put it mathematically when $\frac{d\mu}{dN}$ is large. As $p_0^2 = \frac{4\pi Ne^2}{m}$, we can also take the condition of reflection as $\frac{d\mu}{d(p_0^2)} = \infty$. We shall here consider only the simple case when friction is neglected. The refractive index μ in the absence of friction and polarisation term is given by

$$\mu^2 = 1 - \frac{2p_0^2(p^2 - p_0^2)}{2p^2(p^2 - p_0^2) - p^2 p_T^2 \pm \sqrt{p^4 p_T^4 + 4p^2 p_T^2 (p^2 - p_0^2)^2}} \quad \ldots (1)$$

where, μ = Refractive Index.

p = pulsatance $= 2\pi \times$ frequency.

$p_{L,T} = \frac{eH_{L,T}}{mc}$.

e = charge on an electron.

m = mass of an electron.

$H_{L,T}$ = vertical and horizontal component of the earth's magnetic field.

c = Velocity of light in vacuum.

$$p_0^2 = \frac{4\pi Ne^2}{m}.$$

N = Number of electrons per c. c.

Differentiating with respect to p_0^2 and simplifying a little we get

$$\frac{d\mu}{d(p_0^2)} = -\frac{[2p^2(p^2-p_0^2)-p^2p_T^2 \pm \sqrt{p^4p_T^4+4p^2p_L^2(p^2-p_0^2)^2}]}{[2p^2(p^2-p_0^2)-p^2p_T^2 \pm \sqrt{p^4p_T^4+4p^2p_L^2(p^2-p_0^2)^2}]^{\frac{3}{2}}}$$

$$\frac{(2p^2-4p_0^2)+2p^2p_0^2(p^2-p_0^2)[1 \pm \frac{2p_L^2(p^2-p_0^2)}{\sqrt{p^4p_T^4+4p^2p_L^2(p^2-p_0^2)^2}}]}{[2(p^2-p_0^2)^2-p^2p_T^2 \pm \sqrt{p^4p_T^4+4p^2p_L^2(p^2-p_0^2)}]^{\frac{1}{2}}} \quad \ldots \quad \ldots \quad (2)$$

Now in order that $\frac{d\mu}{d(p_0^2)}$ may be infinitely large either,

$$2p^2(p^2-p_0^2)-p^2p_T^2 \pm \sqrt{p^4p_T^4+4p^2p_L^2(p^2-p_0^2)^2} = 0 \quad \ldots \quad (3)$$

or, $$2(p^2-p_0^2)^2-p^2p_T^2 \pm \sqrt{p^4p_T^4+4p^2p_L^2(p^2-p_0^2)^2} = 0 \quad \ldots \quad (4)$$

Taking the positive sign in equation (4) we get

$$p_0^2 = p^2 \quad \ldots \quad \ldots \quad (5)$$

It is obvious that condition (5) appears to make (2) of the form $\frac{0}{0}$ but when we find the limit of the function as $p \to p_0$ we notice that it comes out to be infinity and hence (5) is a condition of reflection. The negative sign in equation (4) gives,

$$p_0^2 = p^2 + pp_H \quad \ldots \quad \ldots \quad (6a)$$

$$p_0^2 = p^2 - pp_H \quad \ldots \quad \ldots \quad (6b)$$

Again, when we consider equation (3) and take the positive sign, we get the same condition as given by (5) while when we take the negative sign, we get,

$$p_0^2 = \frac{p^2-p_H^2}{p^2-p_L^2}\, p^2 \quad \ldots \quad \ldots \quad (7)$$

Thus we see that this method of looking at the problem gives us not only the usually accepted conditions of reflection as given by equations (5) and (6) but also a new condition represented by equation (7). The equation (5) gives the condition of reflection of what is known as the ordinary ray

and equations (6) and (7) give the conditions of reflection for the extraordinary ray. There is an interesting result from equation (7) that from the same concentration of ionization two waves of different frequencies can be returned and obviously the condition is applicable only to waves whose pulsatance p is either greater than p_H or less than p_L, while condition (6b) is applicable only to waves of pulsatance greater than p_H. We thus come to the conclusion that for waves of pulsatance greater than p_H there is only one concentration of ionization from where the ordinary ray can be reflected while there are, in general, three concentrations from which the extra-ordinary ray can be sent back.

It is easy to see that if we impose a further condition $\mu=0$ upon the general condition of reflection $\frac{d\mu}{d(p_0^2)}=\infty$, we get only one concentration of electrons for the reflection of the ordinary ray and two for that of the extraordinary ray. Thus the general condition of reflection is that at the place of reflection $\frac{d\mu}{dN}$ is very large of which the condition $\mu=0$ is only a special case.

Same condition has been obtained both by Goubau[3] and by Rai[7] proceeding on the assumption that at the place from where radio waves are reflected, group velocity of the incident waves becomes zero.

Quasitransverse propagation

This case shall be obtained from the general case by putting p_L equal to zero and will represent the conditions at the places situated along the magnetic equator. Putting p_L equal to zero in equations (3) and (4) we obtain,

$$2(p^2-p_0^2)^2-p^2p_T^2 \pm p^2p_T^2 = 0 \quad \ldots \quad \ldots \quad (8)$$

$$2p^2(p^2-p_0^2)-p^2p_T^2 \pm p^2p_T^2 = 0 \quad \ldots \quad \ldots \quad (9)$$

The positive sign in both the equations gives,

$$p_0^2 = p^2$$

while the negative sign in (8) gives,

$$p_0^2 = p^2 \pm pp_T$$

and that in (9) gives,

$$p_0^2 = p^2 - p_T^2$$

Thus we see that in this case also we obtain one condition of reflection for the ordinary wave while in general there are three conditions for the exraordinary ray. But if the pulsatance p of the waves is less than p_T, there is only one condition of reflection for extraordinary ray.

Quasilongitudinal propagation

This case shall be obtained by putting $p_T = 0$ in the general case and shall apply to places situated near the magnetic poles. By making the above substitution in equations (3) and (4), we see that (3) does not give any useful result while (4) gives only two conditions of reflection,

$$p_0^2 = p^2 \pm pp_L$$

Thus we see that in this case the results are exactly the same as the ones obtained by the usual method of obtaining the conditions of reflection by putting refractive index equal to zero.

Experimental Results

For the last several months we have been studying the F_2 region. It is generally seen that during the day, except in the early morning when ionization is building up, and in the early part of the night only one of the two magneto-ionic components is visible, (the other being absorbed). Late at night, however, both components are visible and disappear at different frequencies. As already reported[6] on many occasions it was found that if the extraordinary ray disappeared at 4 mc/sec the ordinary one penetrated at about 3·86 mc/sec. Thus the frequency difference was about ·14 mc/sec This result was very perplexing for it could not be explained on the existing theory. But it can be very easily explained on the results obtained in the previous section. It appears that when such low values of critical penetration frequency differences are obtained the extraordinary ray gets reflected from the electron concentration level given by equation (7) while ordinary ray is reflected from the usual level given by equation (5). If f_1 is the critical frequency at any time for the extraordinary ray while f_2 is that for the ordinary ray, using equations (7) and (4) we get,

$$\frac{p_1^2 - p_H^2}{p_1^2 - p_L^2} p_1^2 = p_2^2 \quad \ldots \quad \ldots \quad (10)$$

where $p_{1,2} = 2\pi f_{1,2}$

from which $f_1 - f_2$ for Allahabad comes out to be ·142 mc/sec if f_1 is 4 mc/sec.

Putting condition (10) in the form

$$p_1 - p_2 = \frac{p_T^2}{(1 - p_L^2/p_1^2)\,(p_1 + p_2)}$$, it is clear that as critical frequency increases the difference will decrease and it will show a diurnal variation, being least when the ionization reaches its maximum and greatest when it is minimum while the frequency difference $p_1 - p_2$ obtained by equation (5) and (6) comes out to be a constant quantity approximately $p_{H/2}$ independent of the value of the critical penetration frequency. On some occasions we have also observed a frequency difference of ·64 mc/sec, when the frequency of the waves sent up was about 4mc/sec, This is easily explained from the usually accepted condition of reflection for the extraordinary ray given by equation (6). This value has been reported by investigators all over the world but no mention seems to have been made by any worker of the previous value.

Phase and Group Velocities

Now we will show that the new condition obtained shows that reflection under these circumstances takes place from a place where phase velocity becomes zero. It will also be seen that at this place both phase and group velocities become zero.

The standard formula for group velocity is,

$$\frac{1}{U} = \frac{1}{c}\left(p\frac{d\mu}{dp} + \mu\right)$$

substituting the value of μ and $\frac{d\mu}{dp}$ from equation (1), we have

$$\frac{U}{c} = \frac{N}{D}$$

where, $N = \left[2p^2\,(p^2 - p_0^2) - p^2\,p_T^2 \pm \sqrt{p^4 p_T^4 + 4\,p^2\,p_L^2\,(p^2 - p_0^2)^2}\right]^{\frac{3}{2}}$

$$\left[2(p^2 - p_0^2)^2 - p^2\,p_T^2 \pm \sqrt{p^4\,p_T^4 + 4\,p^2\,p_L^2\,(p^2 - p_0^2)^2}\right]^{\frac{1}{2}}$$

$$\text{and } D = \left[\begin{array}{l} -\left\{ 2p^2 (p^2 - p_0^2) - p^2 p_T^2 \pm \sqrt{p^4 p_T^4 + 4p^2 p_L^2 (p^2 - p_0^2)^2} \right\} 4p^2 p_0^2 \\ +4 p^2 p_0^2 (p^2 - p_0^2) \left\{ 4p^2 - 2p_0^2 - p_T^2 \right. \\ \qquad \left. \pm \dfrac{p^2 p_T^4 + 4p^2 p_L^2 (p^2 - p_0^2) + 2p_L^2 (p^2 - p_0^2)^2}{\sqrt{p^4 p_T^4 + 4 p^2 p_L^2 (p^2 - p_0^2)^2}} \right\} \\ + \left\{ 2p^2 (p^2 - p_0^2) - p^2 p_T^2 \pm \sqrt{p^4 p_T^4 + 4 p^2 p_L^2 (p^2 - p_0^2)^2} \right\} \\ \times \left\{ 2(p^2 - p_0^2)^2 - p^2 p_T^2 \pm \sqrt{p^4 p_T^4 + 4 p^2 p_L^2 (p^2 - p_0^2)^2} \right\} \end{array} \right]$$

Again the phase velocity V is given by

$$\frac{V}{c} = \frac{1}{\mu} = \left\{ \frac{2p^2(p^2 - p_0^2) - p^2 p_T^2 \pm \sqrt{p^4 p_T^4 + 4p^2 p_L^2 (p^2 - p_0^2)^2}}{2(p^2 - p_0^2)^2 - p^2 p_T^2 \pm \sqrt{p^4 p_T^4 + 4 p^2 p_L^2 (p^2 - p_0^2)^2}} \right\}^{\frac{1}{2}}$$

Now it is easy to see that when the reflection conditions given by equations (5) and (6) are fulfilled group velocity is zero and phase velocity is infinite, while when the condition (7) is satisfied both phase and group velocities are zero.

My heartiest thanks are due to Prof. M. N. Saha, D. Sc., F. R. S., and Dr. G. R. Toshnival, D. Sc., for their keen interest and useful discussions. My best thanks are also due to Mr. R. N. Rai for going through the calculations.

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LATTICE ENERGIES OF SOME ALKALI IODIDES AND THE ELECTRON AFFINITY OF IODINE

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Received April 17, 1937

SUMMARY

By studying the dissociation equilibrium of the vapours at high temperatures, the lattice energies of KI and NaI have been determined to be 150·6 and 166·4 K. Cals, respectively. The same experimental arrangement, as described in the work on the electron affinity of chlorine by M. N. Saha and the author has been utilised. By the application of the Born's Cycle to the lattice energy determinations the electron affinity of iodine has been found to be 72. 4 K. Cals. The results are in good agreement with the calculations made from Born's theory and other experimental data.

Introduction

Unlike the other halogens, the electron affinity of iodine has been the subject of investigation of various workers who used different methods. In the present paper an attempt has been made to confirm the older results on the electron affinity of iodine from the measurements of the lattice energies of sodium and potassium iodides. The first investigation on the measurement of electron affinity of the halogens was made by Angerer and Muller [1] who, following an idea of Franck, observed that the halogen gases begin to absorb continuously with a long wave length limit. They attributed the sharp long wave length limit to absorption by X and calculated the electron affinity. Although the results which they obtained are in good agreement with the results obtained from theoretical considerations, yet the interpretation is not free from doubt. A more direct method was used later by J. E. Mayer [9] who determined the lattice energies of the iodides of caesium and potassium and calculated the electron affinity of iodine. In the present investigation the same method has been applied with modified and improved experimental technique. In 1935 P. P. Sutton and J. E. Mayer[13] published another work in which they described a different method for the determination of electron affinity and applied it to the case of iodine. In this method iodine vapour was introduced into an evacuated glass globe in which a tungsten filament surrounded by a concentric anode cylinder was heated. The iodine vapour molecules on coming into contact with the strongly heated filament suffered dissociation into atoms which captured electrons liberated from the filament. The currents due to the negatively charged iodine atoms and electrons could be measured with the help

of the surrounding cylinder. An electromagnetic trap was used to distinguish between the electron and ion currents. Knowing the magnitudes of the ion and electron currents the dissociation constant of the reaction ($Ig_{as} + e = I^- g_{as}$) was determined from which the electron affinity of iodine was calculated. In another method used by G. Glockler and M. Calvin [4] the concentration of iodine ions and electrons was measured from their effect on space charge due to the difference in the masses of the ions and electrons.

On the theoretical side the most important work has been done by Prof. Max Born and J. E. Mayer[1] who have obtained a formula for the lattice energies of ionic crystals by a modification of the older theory. Their formula was used by L. Helmholtz and J. E. Mayer [5] to calculate the lattice energies of the alkali halides, and recently M. L. Huggins[7] has recalculated the lattice energies by taking into account more recent data. In the following table a summary of the results on the lattice energies of alkali-iodides and of the electron affinity of iodine is given.

Table 1

Salt	Lattice Energy (K. Cals.)			Electron affinity of iodine from lattice energy measurements (K. Cals.)			Electron affinity of iodine by other methods (K. Cals.)	
	Theoretical	By direct experiment	By spectroscopic method	Theoretical calculation	By direct experiment	Spectroscopic method	By G. Glockler and M. Calvin	By P. P. Sutton and J. E. Mayer.
LiI	174·1[6] 176·1[7]			75·8[6] 73·1[7]				
NaI	163·9[6] 164·3[7]	166·4*	166	73·9[6] 73·5[7]	72·0*	71·3	74·6 ± 1·5	72·4 ± 1·5
KI	150·8[6] 152·4[7]	153·8[9] 150·6*	151	73·2[6] 71·6[7]	72·62[9] 73·4*			
RbI	145·3[6] 148·0[7]		148	73·8[6] 71·1[7]				
CsI	139·1[6] 142·5[7]	141·5		74·2[6] 70·8[7]				

* PRESENT PAPER

Theory

At high temperatures the process of dissociation of any alkali iodide MI can be represented by the following set of equations :—

$$MI = M + I - D \quad \ldots\ldots (1)$$

$$MI = M^+ + I^- - Q \quad \ldots\ldots (2)$$

$$M = M^+ + e - I_c \quad \ldots\ldots (3)$$

$$I = I + e - E_x \quad \ldots\ldots (4)$$

$$I_2 = 2I - D' \quad \ldots\ldots (5)$$

where D is the heat of dissociation of the alkali halide, Q the heat of dissociation into ions, I_c the ionisation potential of the alkali, E_x the electron affinity of iodine and D′ the heat of dissociation. If K_2 is the dissociation Constant of the reaction (2).
then we have the formnla.

$$\log K_2 = \log \frac{p_{M^+} p_{I^-}}{p_{MI}} = -\frac{Q}{2{\cdot}3RT} + \tfrac{3}{2}\log T + \log\left(1 - e^{-h\nu/KT}\right) + \log\left[\frac{\kappa^{\frac{3}{2}}}{2^{\frac{3}{2}}\pi^{\frac{1}{2}}Ih}\left(\frac{m_M\, m_I^{\frac{2}{2}}}{m_{MI}}\right)\right]$$

Also if L_{MI} be the latent heat of evaporation then the lattice energy $\phi(r_o)$ is given by $\phi(r_o) = Q + L_{MX}$ and $E = D + I_o - Q$. If therefore K_2 is known experimentally at any temperature Q can be determined from which Ex can be calculated.

Experiment

The demountable vacuum furnace used in this experiment has already been described in a previous paper[10], and the internal arrangements have also been described in another communication[11]. The salt was heated in an electric furnace and the vapour was made to enter the high temperature region of the graphite tube where dissociation according to equations (1)—(5) occurred. The products of dissociation passed through a limiting diaphragm and were finally collected by a Faraday cylinder

connected to a cell and galvanometer. The Faraday cylinder was given a small positive or negative potential of 2 volts with respect to the graphite tube in order to measure the negative or positive current. If T is the temperature of the graphite tube, p_{MI} the pressure of the salt, i_{I^-} the current due to the negative ions, the dissociation constant K_2 is given by

$$K_2 = \frac{p_{M^+} \cdot p_{I^-}}{p_{MI}} = \frac{i_{M^+} \cdot i_{I^-}}{e^2 S^2} (2\pi kT) \frac{\sqrt{m_M m_I}}{p_M}$$

where S is the area of the effusion hole, K the Boltz-mann constant, m_M the mass of alkali atom in absolute units and m_I the mass of the iodine atom. The total current l_{M^+} is related to the measured current l_g by the relation

$$i_{M^+} = \frac{2d^2}{r^2} \quad i_G$$

where d is the distance between the effusion hole and the limiting diaphragm.

The temperature of the outer surface of the graphite tube was measured by a disappearing filament type of pyrometer. It was suspected* that this temperature did not give the true equilibrium temperature which is somewhat greater than this and in order to verify this another experiment was performed. A Pt-Rh thermo-couple which was re-calibrated in the laboratory at the melting points of K Cl, Na Cl, and copper was inserted inside the graphite tube, and could be utilised for measuring the temperature of the inner side of the tube. The outer temperature was also measured by the pyrometer. These observations were repeated at several temperatures and a curve was plotted with the outer temperature of the graphite tube against the inner temperature.

*This factor has been neglected in the author's previous works on chlorine[11] and bromine.[13] Those results have been re-calculated now by the author and B. N. Srivastava after making this temperature correction.

Results

In tables 2 and 3 the results obtained for KI and NaI are given. The vapour pressure of KI has been calculated from the empirical formula

$$\log p_{mm} = -\frac{8229}{T} + 8{\cdot}0957$$

due to Fioch and Rodebush[3], and for Na I, von Wartenburg and S-hulz's[14] formula.

$$\log p_{dynes} = -\frac{37000}{4{\cdot}57T} + 11{\cdot}1355$$

has been used.

The magnitudes of the moments of inertia[6] and the characteristic frequency[8] of vibrations* for the two molecules KI and NaI are

I	355×10^{-40}	,	169×10^{-40}
$\frac{h\nu}{k}$	303		408

The lattice energy for KI has been found to be 150·6 K-Cals; and for NaI 166·4 K-Cals. The heats of dissociation of KI and NaI are 76·3 and 72·3 K-Cals respectively and the heats of ionisation of potasium and sodium are 99·5 and 117·3 K-Cals. These data therefore give an average value of 72·4 K-Cals for the electron affinity of iodine.

* The contribution of this in the term $\log (1 - e^{-h\nu/kT})$ was not taken into account in the author's previous works on chlorine and bromine, although it contributes about 7 to 8% in the result. This error was unfortunately balanced in neglecting the temperature correction mentioned before and hence the results were not much affected. The previous results have been recalculated now by the author and B. N. Srivastava

Table 2

Diameter of the effusion hole (mms)	Vapour pressure (dynes/cm²)	Temperature of the outer surface measured by the pyrometer (°C).	Equilibrium temperature obtained from calibration curve	$i_{K^+} \cdot 10^5$ (amps)	$i_{I^-} \cdot 10^5$ amps)	$e^{-\frac{h\nu}{KT}}$	$K_2 \times 10^4$ (dynes)	Q. K ( )	$\phi_{(ro)}$ K-Cals.
1·86	36·19	1600	1680	35·16	26·69	·8563	269·6	101·2	149·4
,,	43·41	1550	1625	21·05	15·00	·8525	72·51	103·4	151·6
,,	52·91	1545	1615	25·07	17·58	·8517	83·7	102·2	150·4
,,	43·41	1445	1510	10·39	7·498	·8437	16·99	102·0	150·2
,,	62·9	1425	1485	9·228	5·768	·8417	16·97	100·6	148·8
1·106	116·3	1550	1625	25·93	9·262	·8525	88·49	102·5	150·7
,,	27·52	1515	1585	·02	3·167	·8494	45·73	101·9	150·1
,,	43·41	1500	1570	6·395	4·541	·8484	52·53	101·4	149·6
,,	90·69	1475	1540	6·275	4·243	·8461	22·68	102·2	150·4
,,	62·9	1450	1515	3·287	2·331	·8441	92·76	104·5	152·7
,,	27·52	1450	1515	2·331	1·673	·8441	10·79	104·0	152·2
,,	62·9	1410	1470	2·749	2·151	·8405	58·09	103·0	151·2

Average of lattice energy = 150·6 K-cal

$E_l = (76·3 + 99·5 - 102·4)$

= 73·4 K-Cals.

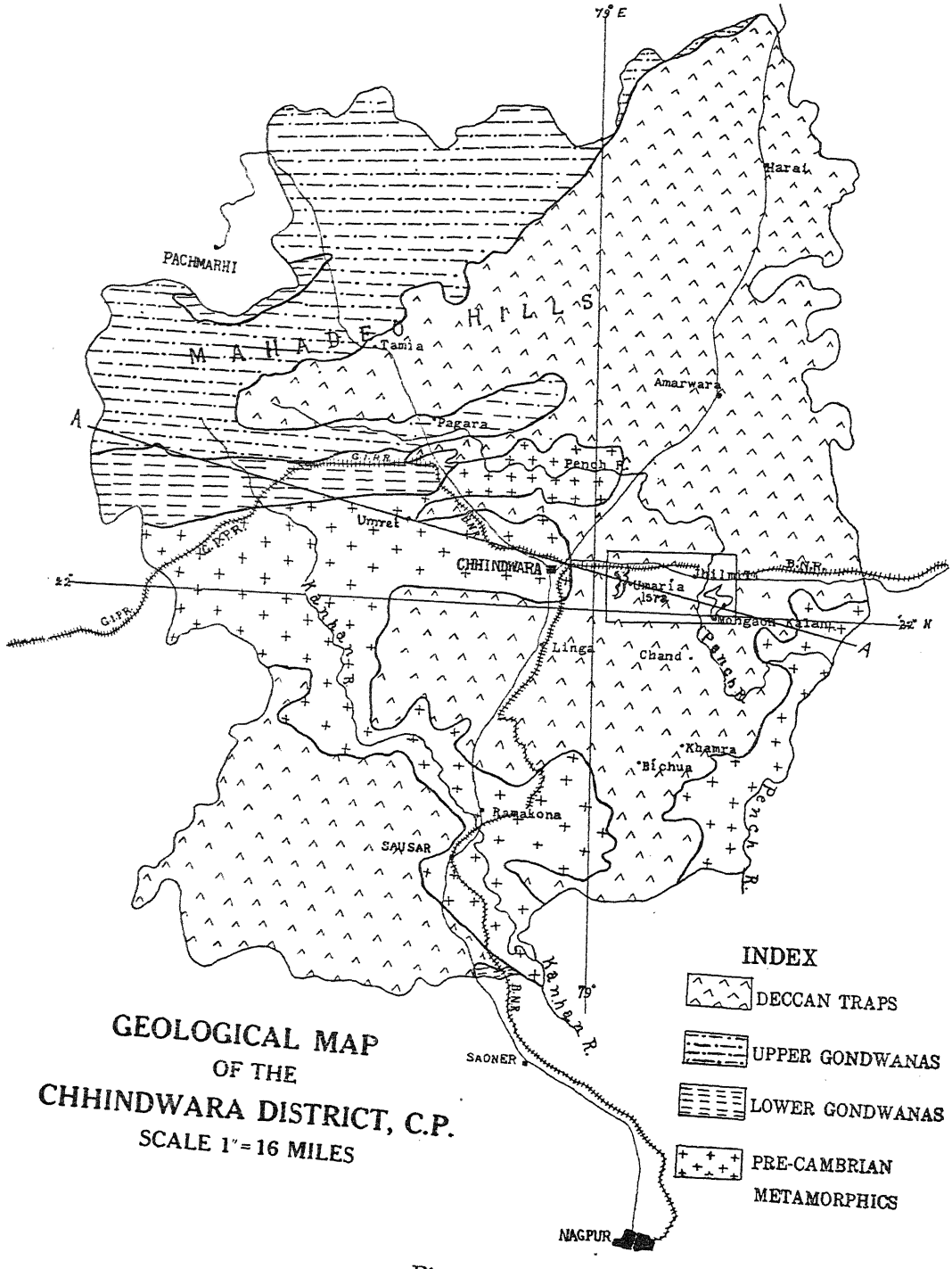
Table 3

Diameter of the effusion hole (mms)	Vapour pressure-dynes/cm.^{2}	Temperature of the outer side of the graphite-tube (°C)	Equilibrum temperature from the calibrated curve (°C)	$i_{Na^+} \times 10^6$ (amps)	$i_{I^-} \times 10^6$ (amps)	$e^{-\frac{h\nu}{KT}}$	$K_2 \times 10^6$	Q. K-Cals	$\phi_{(ro)}$ K-Cals
1·29	32·25	1575	1655	23·43	10·45	·811[illegible]	838·3	120·7	168·3
,,	56·56	1550	1625	24·39	12·44	·8066	188·1	118·5	166·1
,,	18·73	1500	1570	11·71	5·387	·8013	111·9	116·8	164·4
,,	28·21	1480	1550	12·19	5·834	·7995	81·56	116·6	164·2
,,	45·34	1460	1525	10·29	5·404	·7971	38·84	117·8	165·4
,,	114·7	1410	1470	7·508	4·542	·7918	9·156	119·1	166·7
,,	82·13	1410	1470	6·216	3·825	·7918	8·915	119·1	166·7
1·102	67·38	1600	1680	26·3	15·78	·8114	398·9	119·1	166·7
,,	38·68	1570	1650	15·3	10·04	·8050	255·6	119	166·7
,,	23·42	1505	1575	6·23	3·347	·8019	·54·46	119·9	167·5
,,	67·38	1500	1570	9·563	5·26	·8013	46·69	120	167·6
,,	38·68	1475	1540	5·626	3·211	·7986	26·19	119·8	167·4

Average of lattice energy = 166·4 K-Cals.

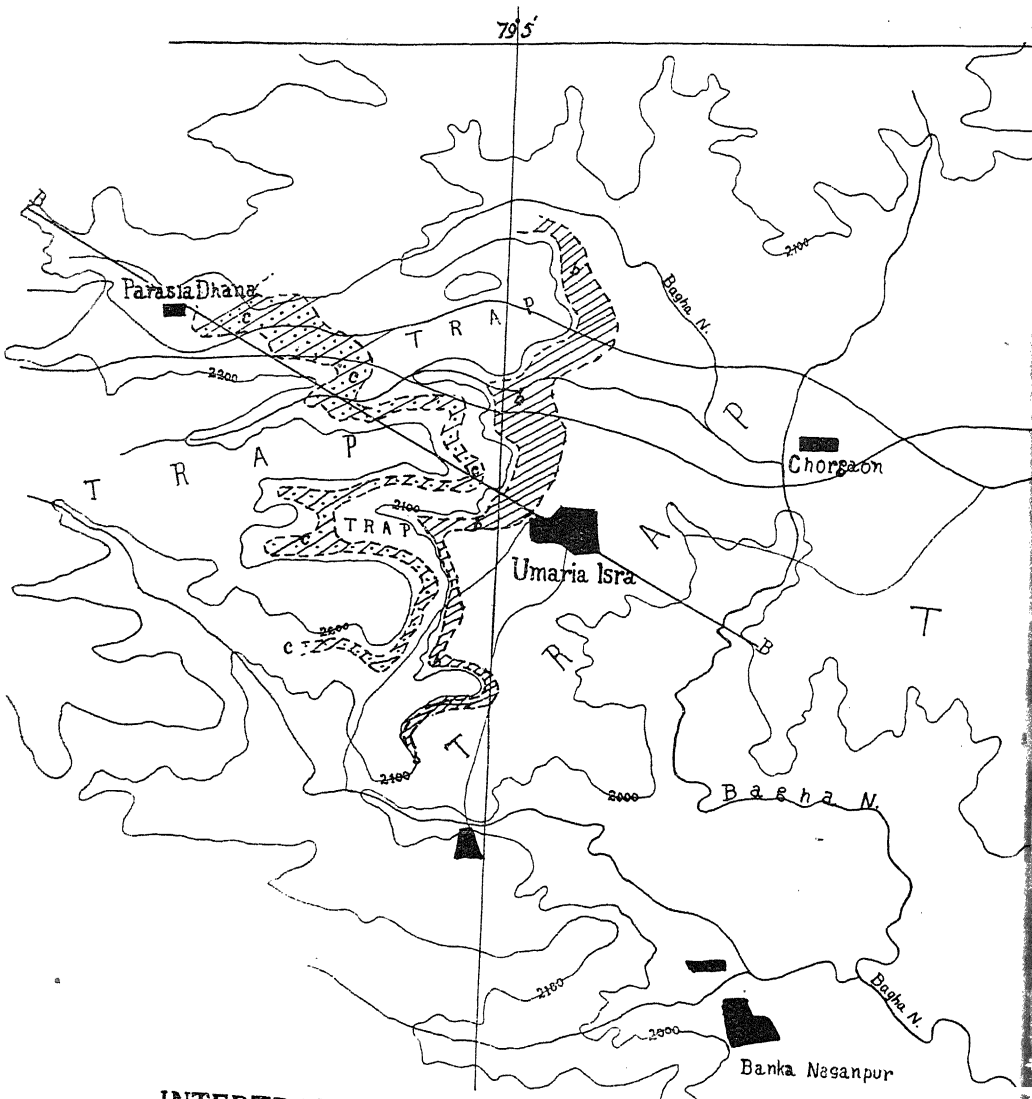
$E_1 = 72·3 + 117·9 - 118·8$

$= 72·4$ K-Cals.



GEOLOGICAL MAP
OF THE
CHHINDWARA DISTRICT, C.P.
SCALE 1" = 16 MILES

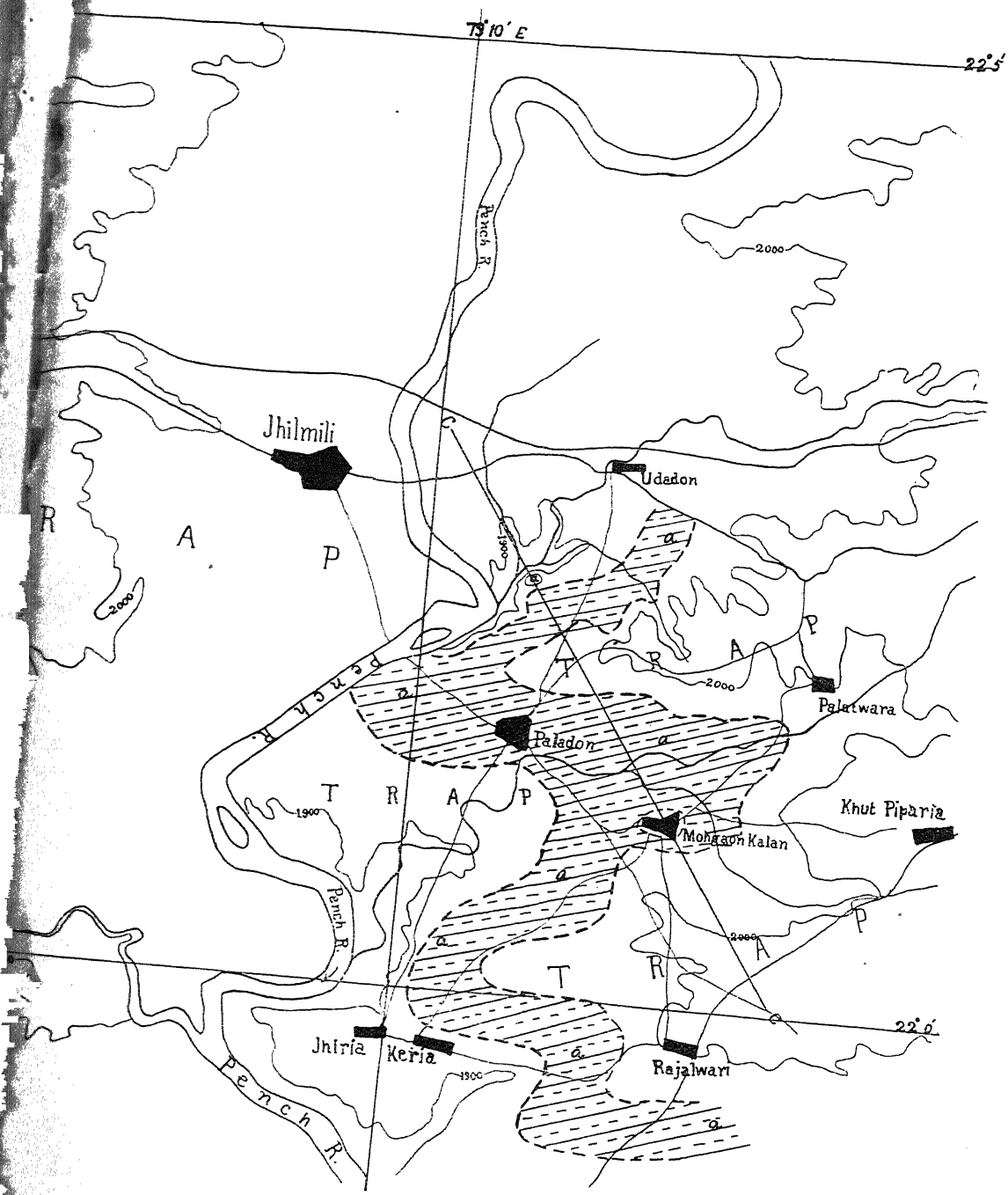
Fig. 1



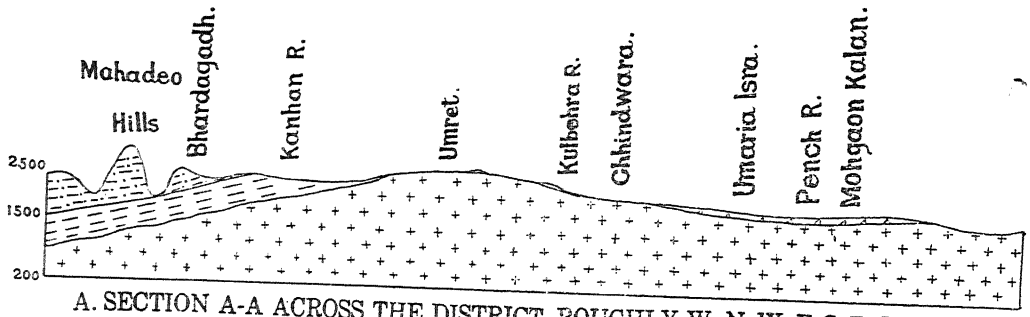
INTERTRAPPEAN BEDS OF
UMARIA ISRA & MOHGAON KALAN

SCALE 1"=4 MILES

Shaded portion represents outcrops of intertrappean horizons. a, b, c,


75°10' E
22°5'
Jhilmili
Udadon
Paladon
Palatwara
Mohgaun Kalan
Khut Piparia
Jhiria
Keria
Rajalwari
Panch R.
2000
1900
T R A P
22°0'

$$= 72{\cdot}4 \text{ K-Cals.}$$

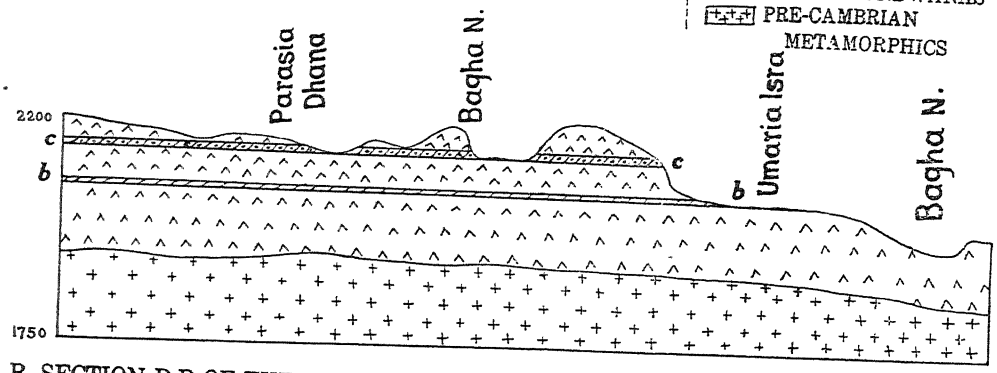


A. SECTION A-A ACROSS THE DISTRICT ROUGHLY W. N. W.-E. S. E. PASSING THROUGH CHHINDWARA TOWN UMARIA ISRA AND MOHGAON KALAN.

HORIZONTAL SCALE 1"=16 MILES

VERTICAL SCALE 1"=4,000 FT.

DECCAN TRAPS
UPPER GONDWANAS
LOWER GONDWANAS
PRE-CAMBRIAN METAMORPHICS

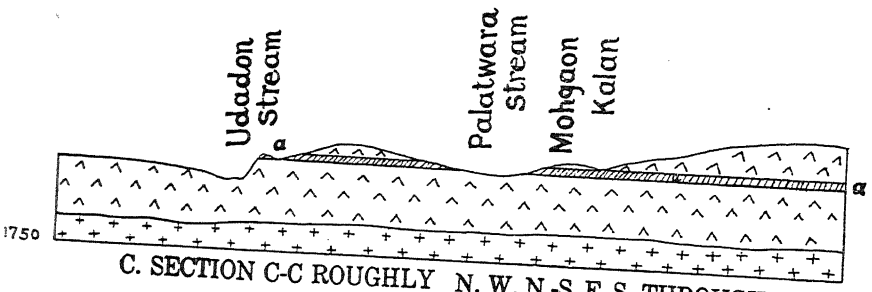


B SECTION B-B OF THE HILLS NEAR UMARIA ISRA

HORIZONTAL SCALE 1"=1 MILES

VERTICAL SCALE 1"=4 00 FT.

C
B
} INTERTRAPPEAN BEDS



C. SECTION C-C ROUGHLY N. W. N.-S. E. S. THROUGH MOHGAON KALAN

HORIZONTAL SCALE 1"=1 MILE

VERTICAL SCALE 1"=4 00 FT.

A INTERTRAPPEAN BEDS

Fig. 3

$= 72{\cdot}4$ K-Cals.

Discussion of results

It can be seen from table 1 that the results obtained are in good agreement with theoretical claculations as well as with the experimental value obtained by other workers. Theoretically the currents due to the positive nad negative ions as given in tables 2 and 3 should be inversely proportional to the square root of their atomic weights, but it will be seen that the magnitude of the current due to I⁻ is always slightly greater than its theoritical value. The same effect was observed in the author's previous work on the lattice energy of potassium and sodium bromides. It was pointed out there that this is due to the presence of free electrons which are thermally liberated from the graphite tube at high temperature In their work on the lattice energies of RbBr and NaCl, L. Helmoltz and J. E. Mayer[6] observed that the experimental ratio of the two currents was different from the theoretical and found an excess of the positive current. According to them the abnormal increase of one particular type of ion current is due to the formation of an ion sheath of the opposite sign round the effusion hole. The experimental conditions under which they have worked are very much different from those in the present work. In the present investigation the order of temperatures used is very high and therefore the former explanation is more probable. It can be shown that probable error in the experiment does not exceed 3 K. cals. The value of the electron affinity of iodine is therefore $72{\cdot}4 \pm 3$. K. cals.

In the end I wish to express my sincere thanks to Prof. M. N. Saha, D.Sc., F.R.S. for his guidance and encouragement throughout the work, and to Mr. B. N. Srivastava for helping in the calibration of thermo-couples and obtaining the curve. Our thanks are also due to the Royal Society of London for giving a grant which enabled us to construct the furnace and buy its accessories.

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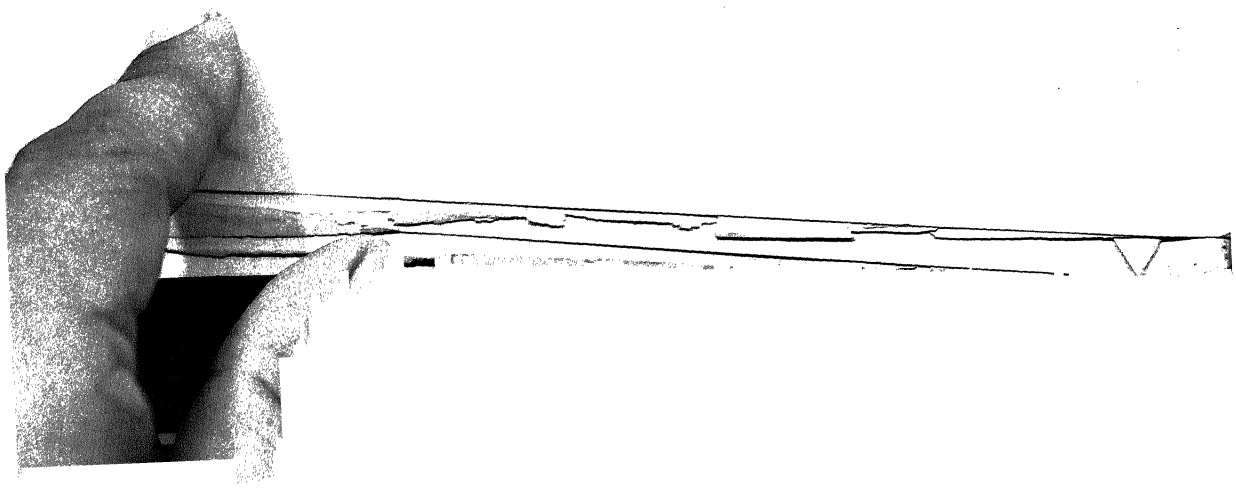

TWO NEW FISH TREMATODES FROM ALLAHABAD

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Communicated by Dr. H. R. Mehra

Received May 6, 1937

SUMMARY

The present paper deals with two apparently new species of fish trematodes belonging to the genera *Fleurogenes* Looss, 1896 and *Opegaster* Ozaki, 1928 which were collected from two different host species, namely, *Callichrous pabda* and *Gobius giuris*, examined at Allahabad.

Pleurogenes pabdai n. sp.

Descriptions of species belonging to this genus from India appear in the works of Lühe (1901), Klein (1905), Mehra and Negi (1928) and Srivastava (1934). These trematodes have been found to parasitize three species of *Rana*. This genus has for the first time been encountered in a fish in this country.

Habitat—Small intestine of *Callichrous pabda*.

Description :—Body elliptical, only slightly flattened dorsoventrally, with a brown patch in the middle due to ripe eggs, and posterior bifid extremity, measures * 1·3—1·4 in length and 0·86 in maximum breadth which lies in the testicular zone. Large scale like cuticular spines are present over the entire body. Subcuticular cells are conspicuous laterally in the preacetabular part and ventrally in the anterior half of the body. The subterminal oral sucker measures 0·2 × 0·25-0·27 in diameter. Prepharynx is present. The pharynx is nearly globular and measures 0·05-0·059 × 0·048-0·059 in size. The narrow oesophagus of about 0·12 length divides into the caeca immediately behind the anterior fourth of the body length.

* All measurements in millimetres.

The intestinal caeca, simple, wide and elongated, extend transversely to the body length terminating just in front of the testes. The acetabulum, pre-equatorial in position, is smaller than the oral sucker and measures 0·18-0·2 in diameter. The genital pore is ventral and lies to the left side near the body wall close behind the pharyngeal level. The excretory pore is situated at the base of the caudal indentation and the bladder is Y-shaped. The main stem of the bladder divides into the cornua at about the middle of the post-acetabular part of the body. The cornua extend up to the middle of the testes lying ventrally to them.

The two large elongated and smooth testes lie symmetrically near the lateral body walls close behind the caeca with their anterior half in level with the acetabulum. The right testis measures 0·3-0·32 × 0·18-0·22 and the left 0·29-0·35 × 0·18-0·2 in size. The cirrus sac is obliquely placed and lies close in front of the left testis with its posterior end immediately in front of the anterior margin of the acetabulum. It is recurved distally before it opens at the genital pore. It contains mostly the large, looped vesicula seminalis which continues terminally into a spherical pars prostatica which in its turn leads into a recurved ductus ejaculatorius. Prostate gland cells are well-developed.

The somewhat oval ovary measures 0·2-0·23 × 0·13-0·18 in size. It is situated to the right side in the oesophageal zone just in front of the testis of that side. The receptaculum seminis, 0·07-0·18 × 0·04-0·09 in size, and Mehlis' gland lie slightly to the right side of the anterior half of the acetabulum. The Laurer's canal opens on the middorsal surface close behind the acetabulum. The long, voluminous, and somewhat regularly coiled uterus occupies all the available space behind the testes and the acetabulum. The descending and the ascending parts lie to the right and the left sides respectively with the connecting loops situated transversely behind the acetabulum. Terminally the uterus lies ventral to the cirrus sac, the metraterm being absent. The ripe eggs measure 0·026-0·029 × 0·011-0·014 in size. The vitellaria, composed of 18—22 large follicles, lie dorsally occupying nearly the entire body region in front of the caeca and extending anteriorly to the middle of the oral sucker.

Remarks:—This species in the extracaecal position of the ovary resembles *P. medians* (Olsson) of Looss, 1894, *P. minus* Pigulewsky, 1931 and *P. japonicus* Yamaguti, 1936 but is separated from all of them as well as other species included in the genus in the bifid posterior end of its body. From *P. medians* it also differs in the position of its transversely placed intestinal caeca, pre-equatorial acetabulum, more anterior position of its ventral genital pore, the shape of its cirrus sac and its continuous vitellaria. It can be clearly separated from *P. minus*, the only other species recorded so far from a fish host, on account of larger body size; different sucker-ratio (acetabulum is larger than oral sucker in *P. minus*); position of intestinal caeca, genital pore, ovary; and shape of the testes. *P. japonicus*

differs from the new species in the position of the acetabulum, marginal genital pore, ovary and testes; in the shape of its cirrus sac; and in the presence of a metraterm.

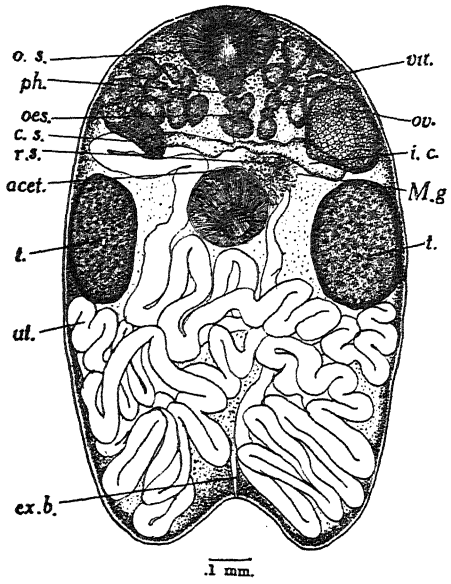

Fig. 1.—Dorsal view of *Pleurogenes pabdai* n. sp.

acet., acetabulum; *a. c.*, anal canal; *c. s.*, cirrus sac; *eg.*, egg; *ex. b.*, excretory bladder; *g. p.*, genital pore; *i. c.*, intestinal caecum; *M. g.*, Mehlis' gland; *oes.*, oesophagus; *o. s.*, oral sucker; *ov.*, ovary; *ph.*, pharynx; *r. s.*, receptaculum seminis; *t.*, testis; *ut.*, uterus; *v. s.*, vesicula seminalis; *vit.*, vitellaria; *vit. r.*, vitelline reservoir.

Opegaster beliyai n. sp.

Harshey (1933, 1937) has described three species of this genus from this country—*O. anguilli* from Allahabad and *O. mastacemballi* and *O. mehrii* from Jubbulpore. He has also given a key of species belonging to this genus in his recent paper. The present species is the fourth representative of this genus from India.

Habitat—Small intestine of *Gobius giuris.*

Description :—The body is flattened, somewhat elongated oval, with pointed anterior and rounded posterior ends and measures 1·28-2·7 in length and 0·45-0·97 in maximum breadth in the acetabular zone. The subterminal oral sucker is 0·11-0·18 in diameter. The prepharynx measures 0·03 in length. The pharynx is 0·07-0·08 long and 0·06-0·09 broad. The oesophagus, 0·11-0·23 in length, bifurcates into the caeca immediately in front of the one-fourth body length from the anterior extremity. The intestinal caeca unite to form an anal canal which opens ventrally a little in front of the posterior end of the body. The prominent acetabulum located

on a short, thick pedicel and with six marginal papillae, three on the anterior border and three on the posterior, measures 0·2-0·32 in diameter. Its centre lies just behind the first third of the body. The genital pore is situated on the ventral surface to the left side behind the middle of the oesophagus nearer the body wall. The excretory pore lies on the dorsal side a little in front of the hinder end of the body in level with the anus.

The transversely elongated testes lie directly one behind the other near the middle of the posterior body half. The anterior testis measures 0·05-0·09 × 0·29-0·34 and the posterior one 0·1-0·13 × 0·25-0·27 in size. The vesicula seminalis extends backwards to the posterior end of the acetabulum. Its terminal part before opening into the small cirrus pouch is surrounded with a few prostate gland cells.

The kidney-shaped ovary lies immediately in front of the anterior testis, measuring 0·07-0·09 × 0·16-0·22 in size. The yolk reservoir is well-developed and situated anterolateral to the ovary. The uterine coils, with 15 eggs, are transversely situated between the ovary and the acetabulum. The ripe eggs are yellow in colour and measure 0·07 × 0·037-0·04 in size. The vitelline follicles begin about the middle of the oesophagus occupying the lateral as well as the post-testicular median fields.

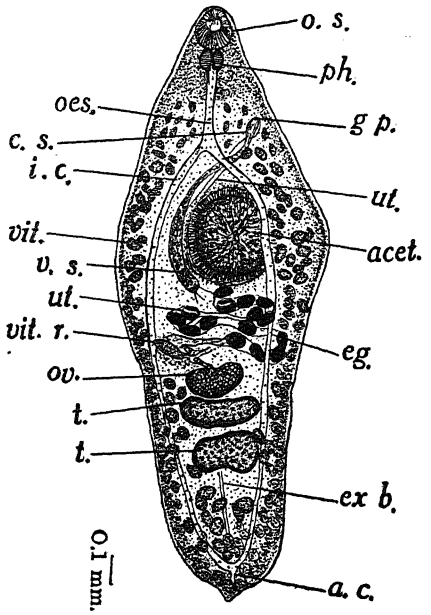

Fig. 2.—Ventral view of *Opegaster beliyai* n. sp. Lettering as in Fig. 1.

Remarks:—In having six marginal papillae on the acetabulum, the present species resembles *O. brevifistula* Ozaki 1928, *O. mehrii* Harshey 1937, *O. mastacemballi* Harshey 1937, and *O. syngnathi* Yamaguti 1934. In *O. brevifistula*

the oesophagus is absent and the acetabular papillae are much more prominent than in my species. *O. mastacemballi* is distinguished from the new species by the presence of a transverse connecting band of vitelline follicles situated close behind the acetabulum. Moreover, the former species is characterised by a shorter oesophagus and the more forward position of the vitellaria which extend up to the middle of the pharynx. *O. syngnathi* has the genital pore slightly to the right side of the intestinal bifurcations, *i.e.*, slightly posterior to the condition in the new species. Besides, in the Japanese species the anterior-most vitelline follicles are rudimentary with the result that the vitellaria appear to begin behind the acetabulum. *O. mehrii* differs from the present form in the position of the genital pore and the absence of anal canal. The description of *O. ozaki* Layman, 1930, is inadequate for assigning it under this genus. According to Yamaguti (1934) the acetabular papillae in *O. ovatus* Ozaki, 1925, reported to be absent, are six in number of which the three anterior ones are conspicuous while the three posterior ones are indistinct. *O. ovatus* is further distinguished from *O. beliyai* by sucker ratio, anterior extent of vitellaria and size of eggs.

The writer is deeply indebted to Dr. H. R. Mehra for his valuable guidance and to Dr. D. R. Bhattacharya for the facilities provided in the Department.

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ON THE CONVERGENCE OF THE CONJUGATE SERIES OF THE DERIVED FOURIER SERIES

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Communicated by Dr. B. N. Prasad

Received January 27, 1937

1. Let the series

$$\tfrac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx) \quad \ldots \quad \ldots \quad (1{\cdot}1)$$

be the Fourier series corresponding to the function $f(x)$ which is periodic with period 2π and integrable in the sense of Lebesgue. The object of this paper is to obtain a very general theorem for the convergence of the series

$$\sum_{n=1}^{\infty} n(a_n \cos nx + b_n \sin nx) \quad \ldots \quad \ldots \quad (1{\cdot}2)$$

which is the conjugate series of the Derived Fourier series corresponding to $f(x)$. It has been shown by Gupta[1] that (1·2) under certain conditions will converge to

$$-\frac{1}{\pi}\int_0^{\pi}\frac{f(x+t)+f(x-t)-2f(x)}{4\sin^2 t/2}\,dt \quad \ldots \quad \ldots \quad (1{\cdot}3)$$

at every point x at which this integral exists. The author proves in this paper that even if (1·3) ceases to exist, the series (1·2) may still converge to

$$-\frac{1}{4\pi}\int_0^{\pi}\psi(t)\,dt - \frac{1}{4\pi}\int_0^{\pi}\Psi(t).\cot\frac{t}{2}\operatorname{cosec}^2\frac{t}{2}\,dt \quad \ldots \quad (1{\cdot}4)$$

where

$$\psi(t) = f(x+t)+f(x-t)-2f(x)$$

and

$$\Psi(t) = \int_0^t \psi(t)\,dt.$$

It has been shown later on that (1·4) can exist even if (1·3) diverges but it will always exist whenever (1·3) is convergent.

2. *Theorem : The series* (1·2) *converges to the sum*

$$-\frac{1}{4\pi}\int_0^\pi \psi(t)\,dt-\frac{1}{4\pi}\int_0^\pi \Psi(t).\ \cot\frac{t}{2}\ \operatorname{cosec}^2\frac{t}{2}\,dt$$

at every point x *at which this integral exists, provided that*

(1) $\psi(t)$ *is absolutely continuous in* $(0, \pi)$,*

(2) $\frac{\psi(t)}{t}$ *is continuous and of bounded variation in* $(0, \pi)$,

(3) $\frac{\psi(t)}{t} = 0(1)$, *for small* t.

Let S_n denote the partial sum of the first n terms of the series (1·2), then we have

$$S_n = \sum_{m=1}^{m=n} m\,(a_m \cos mx + b_m \sin mx)$$

$$= -\frac{1}{4\pi}\int_0^\pi \psi(t)\ \operatorname{cosec}^2\frac{t}{2}\,dt + \frac{1}{4\pi}\int_0^\pi \psi(t)\ \operatorname{cosec}^2\frac{t}{2}\cos nt\,dt$$

$$+\frac{1}{2\pi}\int_0^\pi \psi(t)\,n\cos nt\,dt + \frac{1}{2\pi}\int_0^\pi \psi(t)\cot\frac{t}{2}\cdot n.\sin nt\,dt.$$

$$\equiv -I_1 + I_2 + I_3 + I_4 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2{\cdot}1)$$

We have

$$-I_1 + I_2 = -\frac{1}{4\pi}\int_0^\pi \psi(t)\,dt + \frac{1}{4\pi}\int_0^\pi \psi(t)\cos nt\,dt - \frac{1}{4\pi}\int_0^\rho \psi(t)\cot^2\frac{t}{2}(1-\cos nt)\,dt$$

$$-\frac{1}{4\pi}\int_\rho^\pi \psi(t)\cot^2\frac{t}{2}\,dt + \frac{1}{4\pi}\int_\rho^\pi \psi(t)\cot^2\frac{t}{2}\cos nt\,dt.$$

$$\equiv -\frac{1}{4\pi}\int_0^\pi \psi(t)\,dt + \frac{1}{4\pi}\int_0^\pi \psi(t)\cos nt\,dt + J_1 + J_2 + J_3,$$

where $\rho = \frac{2m+1}{n}\cdot\frac{\pi}{2}$, $2m+1<n$, m being a positive integer and independent of n.

Since $\psi(t)$ is integrable in $(0, \pi)$, by Riemann-Lebesgue theorem, we have

$$\int_0^\pi \psi(t)\cos nt\,dt = 0\ (1) \quad \ldots \quad \ldots \quad (2{\cdot}2)$$

* Gupta,[1] perhaps through oversight, did not mention the condition of absolute continuity of (t) in the statement of his theorem although he made use of this property.

Integrating by parts we get

$$J_1 = -\frac{1}{4\pi}\Psi(\rho)\cot^2\frac{\rho}{2} + \frac{1}{4\pi}\int_0^\rho \Psi(t)\cot^2\frac{t}{2}\, n\sin nt\, dt$$

$$-\frac{1}{4\pi}\int_0^\rho \Psi(t)\cot\frac{t}{2}\operatorname{cosec}^2\frac{t}{2}\, dt + \frac{1}{4\pi}\int_0^\rho \Psi(t)\cot\frac{t}{2}\operatorname{cosec}^2\frac{t}{2}\cos nt\, dt$$

$$= -\frac{1}{4\pi}\Psi(\rho)\cot^2\frac{\rho}{2} + J'_1 + J'_2 + J'_3.$$

Since $\sin nt$ is of bounded variation in $(0, t)$, the total variation being equal to

$$\int_0^t \left| d\sin nt \right| \leqq nt,$$

we may put $\sin nt = P(t) - Q(t)$, where $P(t)$ and $Q(t)$ are positive, monotonic, and non-decreasing functions such that $|P(t)| < nt$ and $|Q(t)| < nt$.

Thus $n\sin t$. $P(t)$ and $n\sin t$. $Q(t)$ are both positive monotonic and increasing functions each being less than n^2t^2.

Now,

$$J'_1 = \frac{1}{8\pi}\int_0^\rho \Psi(t)\cot\frac{t}{2}\operatorname{cosec}^2\frac{t}{2}.\sin t\, n\sin nt\, dt$$

$$= \frac{1}{8\pi}\int_0^\rho \Psi(t)\cot\frac{t}{2}\operatorname{cosec}^2\frac{t}{2}\, n\sin t\, P(t)\, dt$$

$$-\frac{1}{8\pi}\int_0^\rho \Psi(t)\cot\frac{t}{2}\operatorname{cosec}^2\frac{t}{2}\, n\sin t\, Q(t)\, dt$$

$$\equiv L_1 + L_2,$$

where

$$L_1 \leqq n^2\rho^2\int_\xi^\rho \Psi(t)\cot\frac{t}{2}\operatorname{cosec}^2\frac{t}{2}\, dt,\ 0 \leqq \xi \leqq \rho$$

$$= \left[\frac{2m+1}{2}\pi\right]^2\int_\xi^\rho \Psi(t)\cot\frac{t}{2}\operatorname{cosec}^2\frac{t}{2}\, dt = 0(1),$$

when n tends to ∞ and consequently ρ and ξ tend to 0.

Similarly

$$L_2 = 0(1).$$

Therefore

$$J'_1 = 0(1).$$

In a similar manner by expressing cos nt in (o, ρ) as a difference of two monotonic, positive, and non-increasing functions we may prove that

$$J'_3 = 0\,(1).$$

Therefore

$$J_1 = 0(1) - \frac{1}{4\pi}\Psi(\rho)\cot^2\frac{\rho}{2} - \frac{1}{4\pi}\int_0^{\rho}\Psi(t)\cot\frac{t}{2}\operatorname{cosec}^2\frac{t}{2}\,dt \quad \ldots \quad \ldots \quad (2{\cdot}3)$$

Integrating by parts we have

$$J_2 = \frac{1}{4\pi}\Psi(\rho)\cot^2\frac{\rho}{2} - \frac{1}{4\pi}\int_{\rho}^{\pi}\Psi(t)\cot\frac{t}{2}\operatorname{cosec}^2\frac{t}{2}\,dt \quad \ldots \quad \ldots \quad (2{\cdot}4)$$

Now,

$$J_3 = \frac{1}{4\pi}\int_{\rho}^{\pi}\psi(t)\cot^2\frac{t}{2}\cos nt\,dt$$

$$= \frac{1}{4\pi}\int_{\rho}^{\pi}\psi(t)\left[\cot^2\frac{t}{2} - \frac{4}{t^2}\right]\cos nt\,dt + \frac{1}{\pi}\int_{\rho}^{\pi}\psi(t)\frac{\cos nt}{t^2}\,dt$$

$$= 0(1) + \frac{1}{\pi}\int_{\rho}^{\pi}\psi(t)\frac{\cos nt}{t^2}\,dt, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2{\cdot}5)$$

since $\psi(t)\left[\cot^2\frac{t}{2} - \frac{4}{t^2}\right]$ is integrable in $(0, \pi)$.

Hence from (2·2), (2·3), (2·4) and (2·5) we have

$$-I_1 + I_2 = -\frac{1}{4\pi}\int_0^{\pi}\psi(t)\,dt - \frac{1}{4\pi}\int_0^{\pi}\Psi(t)\cot\frac{t}{2}\operatorname{cosec}^2\frac{t}{2}\,dt$$

$$+ \frac{1}{\pi}\int_{\rho}^{\pi}\psi(t)\frac{\cos nt}{t^2}\,dt + 0\,(1) \quad \ldots \quad \ldots \quad (2{\cdot}6)$$

Now $\psi(t)$ being absolutely continuous, let

$$\psi(t) = \int_0^t h(t)\,dt.$$

On integrating by parts, we get

$$I_3 = -\frac{1}{2\pi}\int_0^{\pi}h(t)\sin nt\,dt$$

$$= 0(1), \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2{\cdot}7)$$

since $h(t)$ is integrable in $(0, \pi)$.

Again, integrating by parts we get,

$$I_4 = -\frac{1}{2\pi}\left[\psi(t)\cot\frac{t}{2}\cos nt\right]_o^\pi + \frac{1}{2\pi}\int_o^\pi \frac{d}{dt}\left[\psi(t)\cot\frac{t}{2}\right]\cos nt\,dt$$

$$= 0\,(1) + L,$$

since $\frac{\psi(t)}{t} \to 0$ as $t \to 0$.

Now,

$$L = \frac{1}{2\pi}\int_o^\pi h(t)\cot\frac{t}{2}\cos nt\,dt - \frac{1}{4\pi}\int_o^\pi \psi(t)\,\mathrm{cosec}^2\frac{t}{2}\cos nt\,dt$$

$$= \frac{1}{2\pi}\int_o^\pi h(t)\left[\cot\frac{t}{2} - \frac{2}{t}\right]\cos nt\,dt - \frac{1}{4\pi}\int_o^\pi \psi(t)\left[\mathrm{cosec}^2\frac{t}{2} - \frac{4}{t^2}\right]\cos nt\,dt$$

$$+ \frac{1}{\pi}\int_o^\rho \left[\frac{h(t)}{t} - \frac{\psi(t)}{t^2}\right]\cos nt\,dt + \frac{1}{\pi}\int_\rho^\pi \frac{h(t)}{t}\cos nt\,dt -$$

$$\frac{1}{\pi}\int_\rho^\pi \psi(t)\frac{\cos nt}{t^2}\,dt$$

$$= 0\,(1) + 0\,(1) + 0\,(1) + \frac{1}{\pi}\int_\rho^\pi \frac{h(t)}{t}\cos nt\,dt - \frac{1}{\pi}\int_\rho^\pi \psi(t)\frac{\cos nt}{t^2}\,dt,$$

since each of the following three functions

$$h(t)\left[\cot\frac{t}{2} - \frac{2}{t}\right], \quad \psi(t)\left[\mathrm{cosec}^2\frac{t}{2} - \frac{4}{t^2}\right], \text{ and}$$

$$\frac{d}{dt}\left[\frac{\psi(t)}{t}\right] = \frac{h(t)}{t} - \frac{\psi(t)}{t^2}$$

is integrable in $(0, \pi)$.

Hence $$I_4 = 0\,(1) + \frac{1}{\pi}\int_\rho^\pi h(t)\frac{\cos nt}{t}\,dt - \frac{1}{\pi}\int_\rho^\pi \psi(t)\frac{\cos nt}{t^2}\,dt. \qquad \ldots\ (2{\cdot}8)$$

Therefore from (2·6), (2·7) and (2·8) we have

$$\lim_{n\to\infty} S_n = -\frac{1}{4\pi}\int_o^\pi \psi(t)\,dt - \frac{1}{4\pi}\int_o^\pi \Psi(t)\cot\frac{t}{2}\,\mathrm{cosec}^2\frac{t}{2}\,dt +$$

$$\lim_{n\to\infty}\frac{1}{\pi}\int_\rho^\pi \frac{h(t)}{t}\cos nt\,dt$$

$$= -\frac{1}{4\pi}\int_o^\pi \psi(t)\,dt - \frac{1}{4\pi}\int_o^\pi \Psi(t)\cot\frac{t}{2}\,\mathrm{cosec}^2\frac{t}{2}\,dt,$$

since $\frac{\psi(t)}{t} = \frac{1}{t}\int_o^t h(t)\,dt$ is continuous and of bounded variation and tends to 0 with t, we have by applying de la Vallée Poussin's criterion,

$$\lim_{n\to\infty} \int_\rho^\pi \frac{h(t)}{t} \cos nt\, dt = 0.$$

This completes the proof of the theorem.

3. In order to show that the theorem proved above is more general than that of Gupta, we shall require the following two Lemmas :—

Lemma 1. When the integral

$$\int_o^\delta \frac{\Psi(t)}{t^3} dt \quad \ldots \quad \ldots \quad \ldots \quad (3{\cdot}1)$$

exists, the integral

$$\int_o^\delta \frac{\psi(t)}{t^2}\, dt \quad \ldots \quad \ldots \quad \ldots \quad (3{\cdot}2)$$

will not exist unless $\lim_{t\to o} \frac{\Psi(t)}{t^2}$ *exists.*

If $\epsilon > 0$, we have on integrating by parts

$$\int_\epsilon^\delta \frac{\psi(t)}{t^2}\, dt = \frac{\Psi(\delta)}{\delta^2} - \frac{\Psi(\epsilon)}{\epsilon^2} + 2\int_\epsilon^\delta \frac{\Psi(t)}{t^3}\, dt \quad \ldots \quad \ldots \quad \ldots \quad (3{\cdot}3)$$

From (3·3) it is evident that unless $\lim_{t\to o} \Psi(t)/t^2$ exists, (3·2) will not exist.

Lemma 2. When the integral (3·2) *exists the integral* (3·1) *will always exist.*

It has been shown by Prasad[2] that

$$F(x) = \int_o^x f(t)\, dt = 0\,(x^\delta),$$

where $\delta > 0$, provided

$$\int_o^x \frac{f(t)}{t^\delta}\, dt$$

exists. It follows, therefore, that whenever (3·2) exists,

$$\lim_{t\to o} \frac{\Psi(t)}{t^2} = 0.$$

Hence from (3·3) the lemma follows.

4. We have

$$\int_o^\pi \psi(t)\,\mathrm{cosec}^2\frac{t}{2}\,dt=\int_o^\pi \psi(t)\left\{\mathrm{cosec}^2\frac{t}{2}-\frac{1}{(t/2)^2}\right\}dt+4\int_o^\pi \frac{\psi(t)}{t^2}\,dt$$

$$\equiv A_1+A_2$$

Since $\left\{\mathrm{cosec}^2\frac{t}{2}-\frac{1}{(t/2)^2}\right\}$ is bounded and measurable, the integral A_1 will always exist. Hence the existence of (1·3) is equivalent to the existence of

$$\int_o^\pi \frac{\psi(t)}{t^2}\,dt.$$

Similarly it may be shown that the existence of (1·4) is equivalent to the existence of

$$\int_o^\pi \frac{\Psi(t)}{t^3}\,dt.$$

In view of the lemmas (1) and (2) and 2, it is evident that the theorem of this paper is more general than that of Mr. Gupta.

My thanks are due to Dr. B. N. Prasad for his keen interest in the preparation of this paper.

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DIFFERENTIATION OF A DEFINITE INTEGRAL WITH RESPECT TO A PARAMETER IN CERTAIN CASES WHEN LEIBNITZ'S RULE IS NOT APPLICABLE

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Communicated by Dr. Lakshmi Narayan

Received May 25, 1936

SUMMARY

The author, in this paper, has given rules for differentiation of a definite integral with respect to a parameter in two cases where Leibnitz's rule is not applicable. In the first case, he has considered those functions for which Leibnitz's rule is not applicable owing to the non-existence of the differential coefficient of these functions with respect to the parameter for certain values of the variable; and in the second case functions taken are of such a nature that the necessary and sufficient condition for the validity of Leibnitz's rule is not satisfied, although their differential coefficients with respect to the parameter exist everywhere in the domain.

1. In a previous paper [1] I investigated the necessary and sufficient condition for the validity of Leibnitz's rule for the differentiation of a definite integral with respect to a parameter. In this paper, which is in the nature of a supplement to the above, I will give rules (together with the conditions under which they hold) for the differentiation of a definite integral with respect to a parameter in two cases when Leibnitz's rule is not applicable.

Hobson [2] says "the problem has also been considered, of obtaining the differential coefficient when Leibnitz's rule is not applicable" without giving any reference as to the place and time of its publication. However, so far as my knowledge goes it has not been published anywhere.

2. Let a function $f(x, y)$ of the variables x and y be defined in the interval $a \leq x \leq b$; $c \leq y \leq d$ and satisfying the following conditions:—

(i) It is bounded and integrable (R) with respect to x in (a, b)

(ii) The differential coefficient

$$\frac{d}{dy} f(x, y)$$

exists for all values of y in (c, d) except possibly at the end points where there are only one-sided derivatives and for every value of x in the interval (a, b) except for a set of values of measure zero:

and

(iii) $\frac{d}{dy}\int_a^b f(x, y)\, dx$ and $\int_a^b \left\{ D_y(x, y) \right\} dx$

exist for all values of y in (c, d), where $D_y f(x, y)$ stands for any one of the four derivatives of $f(x, y)$ with respect to y.

Under the above assumptions, the rule for the differentiation of $\int_a^b f(x, y)\, dx$ with respect to y is as follows—

$$\frac{d}{dy}\int_a^b f(x, y)\, dx = \int_a^b \left\{ D_y f(x, y) \right\} dx.$$

and the necessary and sufficient condition for its validity is that corresponding to an arbitrarily chosen positive number ε and to another arbitrarily chosen positive number η a number h_1 such that $|h_1| < \eta$ can be found and also a positive number α such that

$$\left[\sum_{r=1}^{\frac{1}{m}} \frac{(b-a)\, m}{h_1} \left\{ f(a+\overline{b-a}\, rm,\ y+h_1) - f(a+\overline{b-a}\, rm,\ y) \right\} \right]$$

lies between

$$\left[\sum_{r=1}^{\frac{1}{m}} (b-a)\, m \left\{ \overline{D}_y f(a+\overline{b-a}\, r\, m,\ y) \right\} \right] + \varepsilon$$

and $$\left[\sum_{r=1}^{\frac{1}{m}} (b-a)\, m \left\{ \underline{D}_y f(a+\overline{b-a}\ rm,\ y) \right\} \right] - \varepsilon$$

for all values of m in $(0, \alpha)$ which are reciprocal to a positive integer, where $\overline{D}_y$ and $\underline{D}_y$ are the upper and lower derivatives both on the right or both on the left.

Proof :—Divide the interval (a, b) into n equal parts, then since

$\int_a^b f(x,y)\, dx$ exists as an R-integral

$$\frac{d}{dy}\int_a^b f(x, y)\, dx = \lim_{h\to 0} \lim_{m\to 0} \left[\sum_{r=1}^{\frac{1}{m}} \frac{(b-a)\, m}{h} \left\{ f(a+\overline{b-a}\ rm,\ y+h) - f(a+\overline{b-a}\ rm,\ y) \right\} \right]^{4}$$

Now by applying a theorem on repeated limits given by Hobson, it can be shown, as in my previous paper, [4] that the necessary and sufficient conditions in order that

$$\operatorname*{Lim}_{h\to o}\operatorname*{Lim}_{m\to o}\left[\sum_{r=1}^{\overline{m}}\frac{(b-a)\,m}{h}\left\{f\,(a+\overline{b-a}\,rm,\,y+h)-f\,(a+\overline{b-a}\,rm,\,y)\right\}\right]$$

$$=\operatorname*{Lim}_{m\to o}\operatorname*{Lim}_{h\to o}\left[\sum_{r=1}^{\overline{m}}\frac{(b-a)\,m}{h}\left\{f\,(a+\overline{b-a}\,rm,\,y+h)-f\,(a+\overline{b-a}\,rm,\,y)\right\}\right]$$

are

$$(a)\quad\overline{\operatorname*{Lim}_{h\to o}}\left[\sum_{r=1}^{\overline{m}}\frac{(b-a)\,m}{h}\left\{f\,(a+\overline{b-a}\,rm,\,y+h)-f\,(a+\overline{a-b}\,rm,\,y)\right\}\right]-$$

$$\underline{\operatorname*{Lim}_{h\to o}}\left[\sum_{r=1}^{\overline{m}}\frac{(b-a)\,m}{h}\left\{f\,(a+\overline{b-a}\,rm,\,y+h)-f\,(a+\overline{a-b}\,rm,\,y)\right\}\right]$$

converges to zero as $m\to o$ and that

(b) corresponding to an arbitrarily chosen positive number ε and to an arbitrarily chosen number η a number h_1 such that $|\,h_1\,|<\eta$ can be found and also a positive number α such that

$$\left[\sum_{r=1}^{\overline{m}}\frac{(b-a)\,m}{h_1}\left\{f\,(a+\overline{b-a}\,rm,\,y+h_1)-f\,(a+\overline{b-a}\,rm,\,y)\right\}\right]$$

lies between

$$\overline{\operatorname*{Lim}_{h\to o}}\left[\sum_{r=1}^{\overline{m}}\frac{(b-a)\,m}{h}\left\{f\,(a+\overline{b-a}\,rm,\,y+h)-f\,(a+\overline{b-a}\,rm,\,y)\right\}\right]$$

and

$$\underline{\operatorname*{Lim}_{h\to o}}\left[\sum_{r=1}^{\overline{m}}\frac{(b-a)\,m}{h}\left\{f\,(a+\overline{b-a}\,rm,\,y+h)-f\,(a+\overline{b-a}\,rm,\,y)\right\}\right]$$

for every value of m in the interval $(O,\,\alpha)$ which is the reciprocal of a positive integer.

The condition (a) above can be written as

$$\int_a^b \left\{ \overline{D}_y f(x, y) \right\} dx - \int_a^b \left\{ \underline{D}_y f(x, y) \right\} dx = 0$$

where $\overline{D}_y f(x, y)$ and $\underline{D}_y f(x, y)$ are the upper and lower derivatives of $f(x, y)$ with respect to y both on the right or both on the left. This condition is already satisfied as

$$\overline{D}_y f(x, y) = \underline{D}_y f(x, y)$$

except for a set of values of x of measure zero. The condition (b) alone is, therefore, the required necessary and sufficient condition, which can be written in the form given above.

3. Let $f(x, y)$ be a bounded function of x and y $(a \leq x \leq b\,;\, c \leq x \leq d)$ integrable (R) with respect to x then if

$$\frac{d}{dy} \int_a^b f(x, y)\, dx = \int_a^b \left\{ D_y f(x, y) \right\} dx$$

for all values of y $(\neq \alpha)$ in (c, d), α being such that $(c \leq \alpha \leq d)$, $\{D_y f(x, y)$ being any of the four derivatives of $f(x, y)$ with respect to $y\}$, and

$$\lim_{y \to \alpha} \int_a^b \left\{ D_y f(x, y) \right\} dx$$

exists then

$$\frac{d}{dy} \int_a^b f(x, y)\, dx \text{ at } y = \alpha$$

will exist and have the same value as

$$\lim_{y \to \alpha} \int_a^b \left\{ D_y f(x, y) \right\} dx.$$

Proof—

$$\because \lim_{y \to \alpha} \int_a^b \left\{ D_y f(x, y) \right\} dx$$

exists and also

$$\because \int_a^b \left\{ D_y f(x, y) \right\} dx = \frac{d}{dy} \int_a^b f(x, y)\, dx \text{ everywhere except at } y = \alpha.$$

$$\therefore \lim_{y \to \alpha} \frac{d}{dy} \int_a^b f(x, y)\, dx \text{ also exists,}$$

i.e.

$$\lim_{y \to \alpha} \frac{d}{dy} \int_a^b f(x, y)\, dx \text{ on the right}$$

and

$$\lim_{y \to \alpha} \frac{d}{dy} \int_a^b f(x, y)\, dx \text{ on the left,}$$

both exist and are equal.

Therefore the derivatives of $\int_a^b f(x, y)\,dx$ with respect to y at α exist both on the right and on the left and are equal[3]

i.e. $\frac{d}{dy}\int_a^b f(x, y)\,dx$ exists at $y=\alpha$

again $\because \operatorname{Lim}_{y\to\alpha} \frac{d}{dy}\int_a^b f(x, y)\,dx$ exists

therefore $\frac{d}{dy}\int_a^b f(x, y)\,dx$ can have no discontinuity of the second kind. Also, being a differential coefficient, it cannot have a discontinuity of the first kind. Hence it is continuous.

$$\therefore \frac{d}{dy}\int_a^b f(x, y)\,dx = \operatorname{Lim}_{y\to\alpha} \frac{d}{dy}\int_a^b f(x, y)\,dx \text{ at } y=\alpha.$$

$$= \operatorname{Lim}_{y\to\alpha} \int_a^b \left\{ \mathrm{D}_y f(x, y) \right\} dx.$$

4. *Illustrations*:—

(*a*) Let $f(x, y) = y^{1+|x-\frac{1}{2}|} \sin\frac{1}{y}$ when $y \neq 0$

$= 0$ when $y=0$

the function $f(x, y)$ at $y=0$ satisfies the condition given in Art. 2, *i.e.* for given positive numbers e and η a number h_1 ($|h_1| < \eta$) and also α can

be found such that $m+e > \left[\sum_{r=1}^{\frac{1}{m}} \frac{m}{h_1}\left\{ h_1^{1+|rm-\frac{1}{2}|} \sin\frac{1}{h_1} \right\}\right] > -(m+e)$,

for all values of m in $(0, \alpha)$

for $\left[\sum_{r=1}^{\frac{1}{m}} \frac{m}{h_1}\left\{ h_1^{1+|rm-\frac{1}{2}|} \sin\frac{1}{h_1} \right\}\right] = \left[\sum_{r=1}^{\frac{1}{m}} m h_1^{|rm-\frac{1}{2}|} \sin\frac{1}{h_1}\right]$

$$= m \sin\frac{1}{h_1}\left[h_1^{|m-\frac{1}{2}|} + h_1^{|2m-\frac{1}{2}|} + \ldots + h_1^{\left|\left(\frac{1}{m}-2\right)m-\frac{1}{2}\right|} + h_1^{\left|\left(\frac{1}{m}-1\right)m-\frac{1}{2}\right|} + h_1^{\frac{1}{2}} \right]$$

$$= m \sin \frac{1}{h_1}\left[2 \sum_{r=1}^{\frac{1}{2m}-1} h_1^{\,|rm-\frac{1}{2}|} + h_1^{\frac{1}{2}} + 1 \right] \text{ or}$$

$$m \sin \frac{1}{h_1}\left[2 \sum_{r=1}^{\frac{1}{2m}-\frac{1}{2}} h_1^{\,|rm-\frac{1}{2}|} + h_1 \right]$$

according as $\frac{1}{m}$ is even or odd.

$$= m \sin \frac{1}{h_1}\left[2\left(\frac{h_1^{\frac{1}{2}-m}-1}{1-h_1^{\,m}}\right) + h_1^{\frac{1}{2}} + 1 \right] \text{ or}$$

$$m \sin \frac{1}{h_1}\left[2\left(\frac{h_1^{\frac{1}{2}-m} - h_1^{\,\frac{m}{2}}}{1-h_1^{\,m}}\right) + h_1^{\frac{1}{2}} \right]$$

according as $\frac{1}{m}$ is even or odd.

Now if we take m to be such a function of h_1 that $\lim_{h_1 \to o} m = 0$ and $\lim_{h_1 \to o} (m \log h_1) = 0$ $\left(\text{this is quite possible for example } m = \frac{1}{(\log h_1)^2}\right)$ then the limits of the above two expressions as $h_1 \to o$ become

$$\lim_{h_1 \to o} m \sin \frac{1}{h_1}\left\{ 2 \left\{ \frac{h_1^{\frac{1}{2}}(1 - m \log h_1 + A) - 1}{m \log h_1 - A} \right\} + h_1^{\frac{1}{2}} + 1 \right] \text{ or}$$

$$\lim_{h_1 \to o} m \sin \frac{1}{h_1}\left[2 \left[\frac{h_1^{\frac{1}{2}}(1 - m \log h_1 + A) - 1 + \frac{m}{2} \log h_1 - B}{m \log h_1 - A} \right\} + h_1^{\frac{1}{2}} \right]$$

according as $\frac{1}{m}$ is even or odd

where A stands for $\frac{m^2 (\log h_1)^2}{\lfloor 2} - \frac{m^2 (\log h_1)^3}{\lfloor 3} + \ldots\ldots$

and B stands for $\frac{m^2 (\log h_1)^2}{4 \lfloor 2} - \frac{m^3 (\log h_1)^3}{8 \lfloor 3} + \ldots\ldots$

$= 0$ whether $\frac{1}{m}$ is even or odd.

Hence $\frac{d}{dy}\int_o^1 f(x, y)dx = \int_o^1 \left\{ D_y\ f(x, y) \right\} dx$

as a matter of fact both sides of the above equation are zero

(*b*) Let $f(x, y) = \phi_1(x, y) + \phi_2(x, y)$

where $\phi_1(x, y) = y^{1+\left|x-\frac{1}{2}\right|}\sin\frac{1}{y}$ when $y \neq 0$

$= 0$ when $y = 0$

and $\phi_2(x, y) = \left(1+\frac{y}{x}\right)e^{-\frac{y}{x}}$ when $x \neq 0$

$= 0$ when $x = 0$

The above function does not satisfy the condition given in Art. 2 as

$$\sum_{r=1}^{\frac{1}{m}} \frac{m}{h_1} \left\{ \phi_2(rm_1\ h_1) - \phi_2(rm_1\ 0) \right\}$$

cannot be made arbitrarily small by taking m and h_1 small enough.

Hence $\frac{d}{dy}\int_o^1 f(x, y)\,dx \neq \int_o^1 \left\{ D_y\ f(x, y) \right\} dx$

as a matter of fact by actual calculation

$$\frac{d}{dy}\int_o^1 f(x, y)\,dx = -1 \text{ and } \int_o^1 \left\{ D_y\ f(x, y) \right\} dx = 0.$$

(*c*) Let $f(x, y) = \sin\left(4\tan^{-1}\frac{y}{x}\right) - \frac{4xy}{x^2+y^2}\cos\left(4\tan^{-1}\frac{y}{x}\right)$

Then since $\frac{d}{dy}\int_o^X f(x, y)\,dx = \int_o^X \left\{ \frac{d}{dy} f(x, y) \right\} dx$ at $y = 0$

both being equal to $\frac{4X^2}{X^2+y^2}\cos\left(4\tan^{-1}\frac{y}{X}\right)$

and $\lim_{y\to o}\int_o^X \left\{ \frac{d}{dy} f(x, y) \right\} dx$ exists

$$\therefore \frac{d}{dy}\int_o^X f(x, y)\,dx = \lim_{y\to o}\int_o^X \left\{ \frac{d}{dy} f(x, y) \right\} dx \text{ at } y = 0$$

In fact both of them are equal to 4.

[This example has been given by Harnack[1] and Hobson[2] to illustrate the inapplicability of Leibnitz's rule at $y=0$. As shown above, I have indicated a method of finding the differential coefficient in such cases.]

(*d*) The function

$$f(x, y) = \left(1+\frac{y}{x}\right) e^{-\frac{y}{x}} \qquad \text{when} \quad x=0$$

$$= 0 \qquad \text{when} \quad x=0$$

also satisfies the condition given in Art. 3

$$\therefore \frac{d}{dy}\int_o^1 f(x, y)\,dx = \underset{y\to o}{\text{Lim}} \int_o^1 \left\{\frac{d}{dy} f(x, y)\right\} dx \quad \text{at} \quad y=0$$

[This example was given by the author in a previous paper[4] to illustrate the inapplicability of Leibnitz's rule.]

My thanks are due to Dr. Lakshmi Narayan under whose guidance this investigation has been carried out.

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CHEMICAL EXAMINATION OF THE SEEDS OF *PHYSALIS PERUVIANA* OR CAPE GOOSEBERRY, PART II

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Communicated by Dr. S. Dutt.

Received July 1, 1936

SUMMARY

The seeds of *Physalis peruviana* on extraction with benzene yielded 6·23% of an oil of light yellow colour, which is a glyceride of oleic, linolic (unsaturated) and palmitic and stearic (saturated) acids. The unsaponifiable matter consists of a phytosterol $C_{28}H_{44}O_2$. The alcoholic extract showed the presence of saponins and glucose only.

Physalis peruviana (N. O. Solanaceae) called cape gooseberry in English and Makoi in Hindustani, is cultivated in India and hardly differs from *Physalis minima* Linn. except in its large size and more oblong berry. It affords an excellent fruit and is much cultivated in this country. The berries are globular, with slight elongation, about $\frac{3}{4}''$ in length and breadth and having numerous pale yellow tiny seeds embedded in juicy pulp. The seeds constitute 4·5-5·0 per cent of the weight of the husk-free fresh ripe berries. The seeds have been submitted to a detailed chemical examination and, as a result, it is shown that the seeds contain 6·28 % of a pale yellow semi-drying oil which has been worked up in detail as described in the experimental part of the paper. No alkaloid or glucoside could be detected but the presence of an amorphous saponin has been established.

Experimental

The dried seeds of the berries were collected as described in the part I and were ground with great difficulty owing to their small size and hardness. 50 gms. of the crushed seeds were successively extracted with benzene and alcohol in a Soxhlet's apparatus. On subsequent evaporation of the solvent and drying of the extract at 100° to a constant weight the following results were obtained :—

Benzene Extract :—Light yellow oil. Yield 6·23 %.

Alcoholic Extract :—Tasteless light brownish syrupy mass containing some oily or fatty matter and giving a faint greenish coloration with ferric chloride. Yield 3·58%.

The seeds when completely burnt in a porcelain dish left 7·46% of greyish white ash. The ash contained the following positive and negative radicals :—Potassium, sodium, iron, calcium, aluminium, phosphate, silica and chloride.

For complete analysis 2·8 kgs. of the crushed seeds were exhaustively extracted with benzene in a 5 litre extraction flask. The combined benzene extracts on complete distillation of the solvent gave 182 gms. of a pale yellow oil of very thin consistency. The oil-free powder after removal of the traces of benzene was completely extracted with rectified spirit. The combined extract which was tasteless and light brownish yellow in colour was concentrated and then allowed to stand for several days when no crystalline substance separated. It was then repeatedly extracted with hot benzene in order to free it from oil. The alcoholic extract gave a very light green coloration with alcoholic ferric chloride and no precipitate with alcoholic lead acetate. On dilution with water it gave a little flocculent white precipitate which was filtered with great difficulty at the pump and after drying in vacuum desiccator melted at 87°—89°C and was brownish in colour. The amount, being only 0·3 gm., could not be obtained pure.

The diluted alcoholic extract gave no precipitate with lead acetate but a pale yellow precipitate was obtained on addition of a slight excess of basic lead acetate. The precipitate was filtered on the filter pump well washed with hot water and then decomposed in alcoholic suspension with sulphurated hydrogen. The alcoholic filtrate and washings from the resulting lead sulphide deposited globules but no crystalline substance. It gave definite tests for saponins such as frothing on shaking with water, a blue coloration with a solution of potassium ferricyanide and ferric chloride, reduction of mercuric chloride to mercurous chloride, reddish violet coloration with concentrated sulphuric acid.

The filtrate and washings from the basic lead lake was concentrated on the water bath after removal of lead as lead sulphide but did not deposit any crystalline substance even on keeping for several days. It gave an osazone which after crystallisation from alcohol melted at 203°C and was identified as phenyl glucosazone thus proving the presence of glucose.

The crude oil was digested with animal charcoal and filtered through a hot funnel. The purified oil was freed from traces of benzene by heating over water bath and finally by keeping in vacuum desiccator. It is slightly optically active having a small rotation in chloroform $[\alpha]_D^{20} = -38°$. The fatty acids obtained after saponification of the oil are optically inactive, which shows that the rotation is due to the sterol present. The oil burns with a non-sooty flame, and gives positive colour reactions for sterols. The oil is tasteless, semi-drying and free from nitrogen and sulphur. The physical and chemical constants of the oil are $(R.D.)_{27}^{27} = 0·881$; remains clear up to − 10° but becomes thick; acid value = 39·06; saponification value = 179·6; acetyl value = 41·58; unsaponifiable matter = 0·4%; Hehner's value = 93·4; iodine value = 120·5.

150 gms. of the oil were saponified with excess of alcoholic caustic potash and after complete removal of alcohol the resulting soap was freed from alcohol and

then repeatedly extracted with ether in order to remove the unsaponifiable matter. From the resulting soap mixed fatty acids were recovered. The pure mixed fatty acids have $(R.D.)_{23}^{23}=0{\cdot}8826$; neutralisation value=195·8; mean molecular weight =286·1; iodine value=122·45.

100·00 gms. of mixed fatty acids were separated into saturated and unsaturated acids by the Twitchell's[2] Lead-salt-alcohol method as this leads to more quantitative separation than by the lead-salt-ether method (Table I). But during the separation of unsaturated acids, a small amount of resinous acids insoluble in petroleum ether but soluble in ether also separated.

Table I

Acids	% in mixed acids	% in the oil	Iodine value	Mean molecular weight
Saturated ...	13·40	12·52	2·7	286·2
Unsaturated ...	86·60	80·59	134·2	278·7

Elaidin test for the liquid acids:--The liquid acids gave a positive test for elaidic acid.

Oxidation of unsaturated Acids with Potassium permanganate:—10 gms. of the acids were dissolved in aqueous caustic potash and oxidised with a solution of potassium permanganate at room temperature with constant stirring and after the reaction a current of SO_2 was passed to dissolve the precipitated manganese dioxide. The insoluble oxidation product was filtered from it, by successive extraction with ether and boiling water, dihydroxy stearic acid melting at 131-32°C and tetrahydroxy stearic acid (sativic acid) melting at 164-65°C were isolated showing thereby the presence of oleic and linolic acid; hexa-hydroxy stearic acid was entirely absent showing the absence of linolic acid.

The constituents of the unsaturated acids were quantitatively estimated by means of their bromine addition products as recommended by Jamieson and Baughmann.[1] The hexa-bromo-derivative of linolenic acid is insoluble in cold ether, since no precipitate insoluble in ether was formed, the absence of linolenic acid was confirmed. The ether soluble portion was dissolved in petroleum ether and cooled when crystals of linolic tetra-bromide (m. p. 113°C-114°C) were separated showing the presence of linolic acid. The residue was evaporated to dryness and the bromine content estimated. Table III contains the data regarding the analysis of the bromo-derivatives.

The iodine value of the mixture of unsaturated acids was found to be 134·2. The proportion of linolic acid and oleic acid was calculated by the help of the following equations :—

$$X + Y = 100 \quad \ldots \quad (1)$$

$$90{\cdot}07X + 181{\cdot}14Y = 100I \quad \ldots \quad (2)$$

where X and Y are percentages of oleic and linolic acids respectively and I the iodine value of unsaturated acids. The results are given in Table II.

Table II

	% in the unsaturated acids	% in the total fatty acids	% in the original oil
Oleic acid ...	51·55	44·64	41·69
Linolic acid ...	48·45	41·96	39·19

Table III

Wt. of the unsaturated acids taken	5·6920 gms.
Linolic tetrabromide insoluble in petrolium ether ...	3·3712 gms.
Residue (dibromide and tetrabromide)	6·5843 gms.
Bromine content of the residue	32·1%
Dibromide in the residue 72·5% or	4·7720 gms.
Tetrabromide in the residue 27·5% or	2·2780 gms.
Total tetrabromide found	5·6492 gms.
Linolic acid equivalent to tetrabromide	2·6360 gms. or 46·40%
Oleic acid equivalent to dibromide	3·0460 gms. or 53·60%

Table IV gives the percentage of oleic and linolic acids in the unsaturated acids calculated from the above data.

Table IV

	% in the unsaturated acids	% in the total fatty acids	% in the original oil
Oleic acid ...	53·60	46·42	42·37
Linolic acid ...	46·40	40·18	37·53

The theoretical iodine value of a mixture consisting of 53·6 % of oleic acid and 46·42 % of linolic acid is 132·29, which agrees fairly well with the observed iodine value of the unsaturated acids (134·2).

Saturated Acids :—The saturated acids separated by the lead-salt-alcohol method were freed from traces of liquid acids if any by pressing over porous plate and were yellowish white crystalline solid melting at 54-56°C. The pure methyl esters obtained from 10·08 gms. of mixed saturated acid were fractionated under reduced pressure, the boiling points and the pressures being recorded as shown in the Table V and the result of analysis and percentage of acids in the different fractions are given in Table VI.

Table V

Fraction	B. P.	Pressure in mm.	Wt. in gms.
1 ...	169—175°C ..	2 ...	4·4720 ...
2 ...	175—185°C ...	2 ...	2·3296 ...
3 ...	Residue ...	2 ...	2·2944 ...

Table VI

Fraction	Iodine value	Sap. value	Molecular wt.	Unsaturated acids gm.	%	Molecular wt. of esters of saturated acids	Palmitic acids gm.	%	Stearic acids gm.	%	Arachidic acids gm.	%
1	2·3	204·5	274·3	0·0692	1·55	270·9	4·0760	91·12	0·8744	1·96	...	...
2	3·5	195·1	287·6	0·0552	2·36	287·4	0·9184	35·13	1·2788	54·90	...	..
3	9·9	187·5	299·2	0·1528	6·66	299·4	...	...	1·9164	83·60	0·1116	4·42

The percentage of the various acids in the total saturated acids is given in Table VII.

Table VII

Acids	% found in the saturated acids	% in the original oil
Palmitic	52·38	6·56
Stearic	43·38	5·45
Arachidic	1·11	0·14
Unsaturated acids ...	2·97	0·37

Examination of the unsaponifiable matter :—The residue left on concentration of the ethereal extract of the soap was dissolved in 150 c.c. of distilled water and repeatedly extracted with ether. The combined ethereal extracts after a few washings with water was dried over sodium sulphate and ether completely distilled. The crystalline residue on repeated crystallisation from small quantity of rectified spirit was obtained as silky needles melting sharp at 131°C. It gave the various colour reactions of sterols and was optically active, $[\alpha]_D^{23} = -34{\cdot}04°$ in chloroform (c=1·7885). A sample crystallised from chloroform and air dried was combusted. (Found: C=78·40; 78·42; H=10·52, 10·49; $C_{28}H_{44}O_3$ requires C=78·50 %; H=10·28%.) It is a new sterol and the formula given is provisional.

The Authors wish to express their indebtedness to Dr. S. Dutt., D.Sc., P.R.S., for his kind interest in the work.

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CHEMICAL EXAMINATION OF THE SEEDS OF ISABGHOL *PLANTAGO OVATA* FORSKS. PART II. (SUPPLEMENTARY NOTE)

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Communicated by Dr. S. Dutt

Received May 7, 1937

SUMMARY

The saturated acids in the oil from the seeds of Isabghol which are 12·43 % of the total fatty acids and 11·42 % in the original oil contain palmitic acid (32·77 %); stearic acid (60·37 %); and lignoceric acid (6·86 %).

In a previous paper on the "Chemical Examination of the seeds of Isabghol, *Plantago ovata* Forsks" the seeds were subjected to a systematic chemical examination. The investigations revealed the presence of a non-volatile oil, large amount of mucilaginous matter and a quantity of reducing sugars. The oil has been analysed. The composition of the saturated acids could not be determined then. As the investigation is not complete without the examination of the saturated acids, the present note is on the complete and systematic analysis of the saturated acids. The saturated acids from the oil contain palmitic, stearic and lignoceric acids.

Experimental

125 grams of the oil from the seeds were saponified in theu sual way with alcoholic caustic potash and the unsaponifiable matter was separated by ether from the soap solution. The fatty acids liberated from the soap were separated into the unsaturated liquid and saturated solid acids by the Twitchell's lead-salt-alcohol method. About 14 gs. of the saturated acids were obtained from the mixed acids.

Examination of the saturated acids :—The saturated acids separated as above, were freed from traces of liquid acids by pressing them over a porous plate. The acids thus obtained were perfectly solid yellowish white in colour and melted at about 54°. The iodine value was 2·08 and the mean molecular weight calculated from the saponification value (197·4) was 284·2. The mixture of the fatty acids was

then converted into the methyl ester by refluxing 12 grams of the acids with absolute methyl alcohol saturated with hydrochloric acid gas for over 16 hours. The methyl esters (12 gs.) were then fractionally distilled under reduced pressure. The saponification values were determined for different fractions and their mean molecular weights calculated. The free acids were liberated from different fractions and their melting points recorded. The last two fractions had a higher molecular weight and the melting points of the liberated acids probably point to the presence of lignoceric acid. The following tables contain the results of the analysis :—

Table I

Fractions	Boiling points and pressure	Weight of fraction	Saponification value	Mean Mol. Wts. of the esters	Melting points of liberated acids
1	165-75°/1·5 m.m.	2·72 gms.	203·2	276·1	62°-630°
2	175-180°/1·5 m.m.	2·24 gms.	198·4	282·8	65°
3	180-92°/1 m.m. rising up to 200°	1·32 gms.	194·2	289·6	67°-68°
4	200-208°/1 m.m.	3·24 gms.	184·6	304·0	74°-75°
5	Residue (above 210°)	2·48 gms.	176·4	318·1	76°-77°
		12·00			

The iodine value of all the fractions was below 1·5 and therefore no correction is made in the mean molecular weights of the ester.

Table II

Fractions	Esters			Acids			Acids in gms.		
	Palmitic	Stearic	Lignoceric	Palmitic	Stearic	Lignoceric	Palmitic	Stearic	Lignoceric
1	79·36%	20·64%	...	75·23%	18·78%	...	2·05	0·51	...
2	55·52%	44·48%	...	52·64%	40·48%	...	1·18	0·91	...
3	31·31%	68·69%	...	29·70%	62·52%	...	0·39	0·83	...
4	...	93·3%	6·7%	...	84·92%	6·45%	...	2·75	0·21
5	...	76·43%	23·57%	...	69·56%	22·71%	...	1·67	0·55
							3·62	6·67	0·76
							Total	11·05	

Table III

	Acids	In the saturated acids	In the mixed acids	In the original oil
1	Palmitic ...	32·77 %	4·07 %	3·74 %
2	Stearic ...	60·37 %	7·50 %	6·89 %
3	Lignoceric ...	6·86 %	0·86 %	0·79 %
			12·43 %	11·42 %

The author is highly grateful to Dr. S. Dutt of the Allahabad University for his valuable guidance and kind interest during the progress of the work and to the authorities of the Victoria College, Gwalior, for facilities given to him throughout the investigation.

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SIMILARITY BETWEEN NITROGEN FIXATION AND CARBON ASSIMILATION

By N. R. Dhar, E. V. Seshacharyulu and S. K. Mukerji

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Received January 27, 1937

SUMMARY

1. Experimental results show that the nitrogen fixation per gram of carbon oxidised in sun or artificial light is much greater than that in the dark. This happens *in vitro*, in dishes with soils and in fields.

2. The nitrogen fixation when soil is mixed with energy-rich materials, is always greater in light than in the dark, although the Azotobacter count in light is much less than in the dark.

3. It appears that the energy required for nitrogen fixation can be supplied from the energy of the oxidation of energy materials and from sunlight or any other source of light.

4. In presence of glucose, the nitrogen fixation in the soil kept in the dark is highest at 35° but is much less than in light. At higher temperatures, the nitrogen fixation in the dark is much less.

5. Our results show that in nitrogen fixation sunlight or artificial light is used up as in carbon assimilation.

It is well known that most of the photochemical reactions, which have been extensively studied, *e.g.*, between chlorine and hydrogen, chlorine and carbon-monoxide, organic substances and chlorine and bromine, reaction between oxalate and iodine, oxalate and mercuric chloride, etc., are exothermal chemical reactions. The incident light just initiates or starts the chemical changes and may not be necessary for the progress of the reaction. As considerable amounts of energy are set free in these reactions, there is no proportionality between the number of molecules decomposed or formed and the number of quantum of light absorbed by the reacting systems and in the majority of these reactions Einstein's law of photochemical equivalence fails, as many molecules are formed or decomposed per quantum of light absorbed.

On the other hand, the photosynthesis of energy materials on the leaf surface according to the equation

$$6\,CO_2 + 6\,H_2O + 676\,Cal = C_6H_{12}O_6 + 6O_2$$

is the most important endothermal chemical reaction, which has been studied by chemists, agriculturists and plant physiologists for over hundred and fifty years. This reaction apparently cannot proceed in the absence of light, which supplies the energy required in this highly endothermal reaction.

For over twentyfive years in numerous publications from this laboratory, we have been investigating the initiation of exothermal reactions by the energy obtained from another exothermal reaction. Thus we have shown that the energy derived from the oxidation of an aqueous solution of oxalic acid by potassium permanganate, can form a large amount of calomel from a mixture of mercuric chloride and oxalic acid even in the dark. Similarly the oxidation of different organic acids by potassium permanganate, manganese dioxide, persulphuric acid, nitrous acid, iron salts and hydrogen peroxide and other oxidizing agents can induce or start several chemical reactions. Moreover, the oxidation of sodium sulphite or ferrous hydroxide or cerous hydroxide or manganous hydroxide, etc., by air leads to the oxidation of nickelous hydroxide or carbohydrates, fats, proteins, etc, even in the absence of light. The energy materials like carbohydrates, fats, and proteins have also been oxidised simply by passing air through their solutions or suspensions exposed to light. In all these cases exothermal reactions have been initiated either through the agency of light or the energy available from another exothermal chemical reaction.

Recently we have been successful in inducing an endothermal reaction $N_2+O_2+43{\cdot}2$ Cal$=2NO$ by the energy obtained from the oxidation of different carbohydrates by air. Thus Dhar and Mukerji [2, 6] have obtained four to six milligrams of nitrogen fixation per gram of canesugar or glucose oxidised (*e.g.*, about 12 milligrams of nitrogen fixed per gram of carbon oxidised) by passing sterile air through solutions of the carbohydrates containing ferrous hydroxide or cerous hydroxide or sterile soil in the complete absence of bacteria in the dark. Recently Dhar and Sundara Rao have obtained nitrogen fixation of 9 milligrams per gram of canesugar oxidised (*e.g.*, about 23 milligrams of nitrogen fixed per gram of carbon oxidized) in the sunlight by passing air through the solutions of the carbohydrate in contact with ferrous or cerous hydroxide. These experimental results show definitely that per gram of energy material oxidised, the nitrogen fixation in sunlight is practically double of that obtained in its absence.

We have carried on numerous experiments on nitrogen fixation when soil mixed with energy materials are exposed to light or kept in the dark. These experiments have been carried on with soils kept in dishes and with field soils. When 20 tons of molasses containing 60% carbohydrates are added per acre of normal soil exposed to air and sunlight which is turned over once every month, 493 lbs. of nitrogen are fixed. Using 40 tons, 605 lbs. of nitrogen are fixed, whilst with 3 tons and 10 tons of molasses per acre, the nitrogen fixations are 112 lbs. and 270 lbs. respectively. Carbohydrate manuring in tropical countries appears to be better than symbiotic nitrogen fixation, which adds 50—100 lbs. nitrogen per acre.

The following results on nitrogen fixation have been obtained with soils kept in light and in the dark.

Table I

Nitrogen fixed per gram of carbon oxidised

Experiments in dishes

Canesugar + Ca CO_3	(Exposed)	15·8	mgm.
Canesugar	„	14·7	„
Canesugar + Ca CO_3	(Dark)	10·5	„
Canesugar	„	10·2	„
Glucose + Ca CO_3	(Exposed)	12·5	„
Glucose	(Exposed)	12·5	„
Glucose + Ca CO_3	(Dark)	6·5	„
Glucose	(Dark)	6·5	„
Mannitol	(Exposed)	12·8	„
Mannitol	(Dark)	6·9	„
Starch	(Exposed)	7·58	„
Starch	(Dark)	3·59	„
Glycerol	(Exposed)	6·04	„
Glycerol	(Dark)	3·1	„
Cowdung	(Exposed)	28·3	„
Cowdung	(Dark)	13·6	„

Field Trials

Plot 6′ × 4′ containing 5 Kg. of molasses.	(Exposed)	22·9	„
	(Dark)	13·2	„
Plot 6′ × 4′ containing 10 Kg. of molasses.	(Exposed)	17·8	„
	(Dark)	10·06	„

Exactly similar results have been obtained with butter, hay and other energy materials.

The following results were obtained by exposing soil and canesugar to light from a 1000 watt filament lamp.

Table II

Experiments in dishes at 40°. Nitrogen fixed per gram of carbon oxidized

1% Canesugar	(Exposed)	33	milligrams
1% Canesugar	(Dark)	6·2	„
2% Canesugar	(Exposed)	23·0	„
2% Canesugar	(Dark)	6·8	„
3% Canesugar	(Exposed)	20	„
3% Canesugar	(Dark)	5·5	„
5% Canesugar	(Exposed)	14	„
5% Canesugar	(Dark)	5·4	„

The foregoing results definitely show that nitrogen fixation in sunlight or artificial light is much greater than that obtained in the dark per gram of the energy material oxidised. As the amount of energy available from the oxidation of one gram of energy material is the same both in the light and the dark, it is expected that the nitrogen fixation should be the same in light as in the dark, but the experimental results show that with all the energy materials, the nitrogen fixation for the same amount of energy output from the oxidation of the energy material, in light is much more than in the dark. Hence we are led to conclude that in nitrogen fixation, which is an endothermal reaction, the radiations of the sun or from other sources are actually utilised in effecting more nitrogen fixation as in photosynthesis or carbon assimilation in plants or in the formation of ozone by light absorption.

In a recent communication to "NATURE," Dhar and Mukerji[5] have reported that when soil is mixed with cowdung and exposed to air and light there is considerable nitrogen fixation. Our results show that with cowdung as an energy material the nitrogen fixation in the fields receiving sunlight is much greater than in the dark when compared per gram of carbon oxidized.

Dhar and Seshacharyulu have determined the numbers of Azotobacter in soils containing energy materials and the amount of nitrogen fixed in light and dark and they have shown that the number of Azotobacter per gram of soil kept in light is much smaller than in the soil kept in the dark although the nitrogen fixation in light is much greater than in the dark. The following tables show some typical results obtained in this direction.

Table III

1 kilogram of soil + 50 gms. starch. Dishes exposed to sunlight daily for 8 hours. (Temperature 36° - 48°)

Date of analysis	NH_3N	Total -N	Total -C	Azotobacter in millions per gram of dry soil	
10-3-1936	0·001%	0·042%	0·441%	7·2	
9-4-1936	0·001	0·042	.....	6·8	
30-4-1936	0·0018	0·0433	2·4425	8·2	
20-5-1936	0·0023	0·0442	2·37	9·1	
20-6-1936	0·0029	0·0451	2·2765	15·6	Nitric nitrogen varied from 0·0024 % to 0·00336 %
16-7-1936	0·0033	0·0461	2·1753	19·8	
28-7-1936	0·0037	0·0472	2·0932	28·2	
7-10-1936	0·0052	0·051	1·4411	38·6	
7-11-1936	0·0056	0·053	1·2592	31·2	
7-12-1936	0·0058	0·0538	1·0181	39·4	
2-1-1937	0·006	0·0555	0·7217	32·8	
6-2-1937	0·00464	0·056	0·5953	28·5	

Nitrogen fixed per gram of carbon oxidized in sunlight = 7·58 milligrams.

Dishes kept in the dark.
(Temperature 30° – 38°)

Date of analysis	NH_3N	Total -N	Total -C	Azotobacter millions per gram of dry soil	
12-3-1937	0·004	0·0547	0·5684	16·5	
11-4-1937	0·00356	0·053	0·5486	9·5	
10-3-1936	0·001 %	0·042	0·441	7·2	
6-4-1936	0·0010	0·042	... ...	8·1	Nitric nitrogen varied from 0·0024% to 0·00264 %
30-4-1936	0·0014	0·0420	2·5682	11·2	
20-5-1936	0·0015	0·0420	2·4761	18·5	
20-6-1936	0·0018	0·0433	2·4137	26·8	
16-7-1936	0·0021	0·0437	2·3352	46·0	
28-7-1936	0·0023	0·0442	2·2654	98·6	
7-10-1936	0·0031	0·0461	1·8286	205·0	
7-11-1936	0·0032	0·0461	1·7164	265·0	
7-12-1936	0·0032	0·0466	1·5382	305·0	
2-1-1937	0·0035	0·0472	1·3124	365·5	
6-2-1937	0·0032	0·0477	1·0538	380·0	
2-3-1937	0·003	0·0482	0·7854	385·0	
11-4-1937	0·0024	0·0482	0·5368	300·0	

Nitrogen fixed per gram of carbon oxidized in the dark = 3·13 milligrams.

From the foregoing observations it is clear that nitrogen fixation can take place in the dark when the soil is supplied with energy obtained from the oxidation of energy materials, like carbohydrates, pentosans, cellulose etc., but the nitrogen fixation is considerably increased when these systems also receive sunlight or artificial light, which is utilized in this process as in photosynthesis in the plant kingdom or in the formation of ozone by light absorption.

Recent experiments of Dhar and Seshacharyulu show that the nitrogen fixation in soil fed with 2 % glucose in the dark is 7·76 milligrams per gram of carbon oxidized at 35°. At higher or lower temperatures the nitrogen fixation in the dark is much less, whilst under identical conditions the nitrogen fixation in sunlight is 13·1 milligrams per gram of carbon oxidized kept at a temperature of 42°. Hence light and not increase of temperature is responsible for increased nitrogen fixation in light.

Many plant physiologists have reported that there is an intimate relationship between photosynthesis and respiration in plants. Thus Plester[10] has stated that while it is evident that there is no parallelism between chlorophyll-content and rate of photosynthesis, the quotient $\frac{\text{Respiration}}{\text{Photosynthesis}}$ for the light green varieties seems fairly constant, as is evident from the following results :—

Ptelea = 1·77, Catalpa = 1·72, Mirabilis = 2·0,
Ulmus = 2·0, Populus = 2·1.

Henrici,[8] Boysen – Jensen[1], Spoehr and McGee[11] and others have found that plants, which have a high rate of respiration, have also a high rate of carbohydrate formation.

Dhar and Atma Ram[3] have postulated that in nature the photosynthesis that is taking place in the plants by the absorption of sunlight is aided by the energy obtained in plant respiration, which goes on as long as the plant lives.

When solutions of organic substances are exposed to light and air they are oxidized photochemically to carbondioxide and water along with the formation of formaldehyde. The following results have been obtained by Dhar and Atma Ram :—

Table IV

Comparative experiments in the photosynthesis of formaldehyde *in vitro* from sodium salts of fatty acids, carbohydrates, proteins and potassium carbonate solutions exposed to sunlight in quartz vessels.

Temperature 35°, volume of solution exposed=25c. c. for 6 hours

System exposed	Amount of substance decomposed or photo-oxidised in gram moles.	Percentage of the substance oxidised or decomposed	Amount of formaldehyde formed in gram moles.	Ratio of amount of substance oxidised or decomposed to that of formaldehyde formed
M/5 $KHCO_3$	0·098	4·9	0·000001	9800
M/100 Sodium oleate.	0·00011	1·1	0·0000037	29
M/100 Sodium palmitate.	0·000091	0·9	0·000003	30
M/100 Sodium stearate.	0·000082	0·82	0·0000023	36
M/100 Cane-sugar.	0·00015	1·5	0·0000018	83
M/100 Glucose	0·00016	1·6	0·0000017	126
M/100 α-alanine	0·00021	2·1	0·0000018	117
M/100 Aspartic Acid.	0·00018	1·8	0·0000015	120

The foregoing results show that the amount of formaldehyde formed by exposing potassium bicarbonate solutions to sunlight in quartz vessels is smaller than the amount formed in the photo-oxidation of the organic substances. Moreover, when we compare the amounts of formaldehyde formed with the number of molecules of bicarbonate decomposed or the organic substance oxidised, a great difference is at

once observed. Although the salts of the fatty acids are oxidised to a smaller extent than the carbohydrates and the amino acids under comparable conditions, the amounts of formaldehyde produced are greater in the case of the salts of the fatty acids than with carbohydrates and amino acids.

It seems likely that the energy generated in the photo-oxidation of these organic substances supplies a part of the energy necessary for the photoformation of formaldehyde. We are of the opinion that in nature, the photosynthesis that is taking place in the plants is aided by the energy obtained in plant respiration, which goes on as long as the plant lives. The ease with which formaldehyde or other energy-rich compounds are formed in plants is partly due to their getting a constant supply of energy from the oxidation of the food materials present in the plant. We have postulated that the most important chemical change in the formation of carbohydrates in plants and in the formation of formaldehyde in nature from carbon dioxide and water is the photolysis of water into H and OH. The amount of energy required to decompose a gram molecule of water into H and OH is approximately the same as that necessary for the formation of a gram mole of formaldehyde from carbon dioxide and water. These are highly endothermal changes requiring radiations of wavelength 2552Å (112 Calories). In nature, however, photosynthesis takes place in visible light especially the red. We are of the opinion that the energy derived from respiration in the plants already supplies a part of the energy necessary for the photosynthesis and thus renders the photodecomposition of water possible by longer wavelengths. Although the adsorption of carbon dioxide and water by the chlorophyll of the leaf may partially activate these substances just as the adsorption of hydrogen and oxygen on a platinum or palladium surface renders them active, it appears to us that this activation of carbon dioxide and water by their adsorption on the leaf surface is less important than their activation by the adsorption of energy from respiration.

There is an intimate relation between respiration and photosynthesis in the plant kingdom, because photosynthesis proceeds with difficulty without the energy available from respiration for the partial activation of carbon dioxide and water vapour. The need of the presence of oxygen in photosynthesis is also explained from the same point of view.

It is easier to obtain formaldehyde or any other energy-rich compound from carbonic acid and bicarbonate solution on exposure to light when a suitable exothermal reaction is taking place in the system along with the photosynthetic reaction.

It is well known that large amounts of energy are liberated in the oxidation of carbohydrates

$$C_6H_{12}O_6 + 6O_2 = 6CO_2 + 6H_2O + 676 \text{ Cal.}$$

This energy can be utilised in bringing about the endothermal reaction between nitrogen and oxygen ($N_2 + O_2 + 43{\cdot}2 = 2\ NO$) on the soil surface even in absence of sunlight. The oxide of nitrogen in contact with the soil moisture and lime can cause the formation calcium nitrate, which may be reduced to ammonium salts by the excess of carbohydrates added to the soil. Moreover the energy liberated in the oxidation of carbohydrates on the soil surface may lead to the decomposition of water molecules according to the following equation :—

$$H_2O + 112\ Cal = H + OH.$$

The atomic hydrogen can readily combine with molecular nitrogen forming ammonium salts on the soil surface. When soil mixed with carbohydrates, glycerol, etc., comes in contact with air, the carbon decreases and the total and ammoniacal nitrogen increase by nitrogen fixation. By light absorption, all these processes, which can take place in the dark provided the energy of the oxidation of the carbohydrates is available on the soil surface, are accelerated and hence more nitrogen fixation per gram of energy material oxidized is observed in light. Under natural conditions large amounts of nitrogen are fixed in the soil with the energy of the oxidation of celluloses aided by sunlight.

According to Moore[9] (1921) sunlight causes a slight union of nitrogen and oxygen in the air, resulting in the formation of oxides of nitrogen. Dhar and Sanyal[7] (1925) have observed the formation of traces of nitrous acid when air freed from impurities is bubbled through conductivity water in presence of ultraviolet light. This reaction cannot go far, as nitrous acid decomposes readily in light according to the equation

$$3\,HNO_2 \rightleftarrows HNO_3 + 2\,NO + H_2O$$

Moreover nitric acid and nitrates also decompose in light. Similarly the following reaction $CO_2 + H_2O + 112$ Cal. $= HCHO + O_2$ can take place to a very limited extent in air as formaldehyde is readily decomposed in light. Dhar and Atma Ram obtained 0·001 gram formaldehyde per litre of freshly collected rain water. The combinations of nitrogen and oxygen and carbon dioxide and water vapour are greatly accelerated when, along with sunlight, the energy of the carbohydrate oxidation is available to the systems.

The efficiency of nitrogen fixation is also low as in the case of carbon assimilation. It is well known that in many cases, only 0·1 to 0·2 % of the light energy is utilized in carbon assimilation in plants. In our experiments with nitrogen fixation, not even 1% of the energy liberated by the oxidation of the carbohydrates or cellulosic materials is utilized in fixation.

N.B.—As the publication of the paper was delayed for a long time, a few minor additions and alterations had to be made.

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NITROGEN FIXATION AND AZOTOBACTER COUNT ON THE APPLICATION OF CARBOHYDRATES AND OTHER ENERGY-RICH MATERIALS TO THE SOIL, PART III

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Communicated by Prof. N. R. Dhar, D.Sc, I.E.S.

Received February 24, 1937

SUMMARY

1. The influence of sunlight on nitrogen fixation and Azotobacter count in the soils mixed with energy-rich materials has been studied.

2. Nitrogen fixation is always greater in the exposed soils than in the soils kept in the dark.

3. When calculated the nitrogen fixed per gram of carbon oxidised is practically double in the exposed soils than in the dark.

4. Light plays an important part in the nitrogen fixation in the soils when supplied with energy materials.

5. Even in the field soils nitrogen fixation takes place on the addition of starch and glycerol.

In a previous paper[2] it has been reported that the nitrogen fixation with soils containing carbohydrates like canesugar and glucose in basins is always greater when exposed to sunlight daily for about 8 hours than in basins kept in dark although the Azotobacter count is very much greater in the dark basins than in the light.

I have extended my experiments to mannitol, dextrin, maltose, fructose and galactose as energy materials. The experimental procedure is the same as recorded in the Part II.

In order to see whether nitrogen fixation takes place in field conditions I have applied definite amounts of starch and glycerol to small plots of land.

Three plots of equal area (4′ by 4′) were taken. To one plot 2 kilograms of glycerol and to the other 2 kilograms of starch were applied. Another plot was kept as control. All these plots were dug up and watered regularly to keep an uniform moisture content. At regular intervals the Azotobacter count and nitrogen estimations of these plots were carried on.

The following are the results obtained :—

Table I

1 kilogram of soil + 20 gms. Mannitol exposed (Temp. 38°—48°)

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. of dry soil	
20-4-1936 original soil	0·00152	0·043	0·494	6·5	Nitric nitrogen varied from 0·00308 % to 0·00358 %.
5-5-1936	0·002	0·043	1·2087	11·8	
22-5-1936	0·0028	0·0454	1·0725	16·6	
8-6-1936	0·0035	0·0473	0·9539	20·5	
24-6-1936	0·00436	0·0489	0·8118	28·5	
14-7-1936	0·005	0·0507	0·7156	32·5	
26-7-1936	0·00536	0·051	0·6432	29·8	
10-8-1936	0·0052	0·051	0·5846	30·5	
12-10-1936	0·00424	0·0507	0·5124	18·5	

Table II

1 kilogram of soil + 20 gms. Mannitol dark (Temp. 30°—38°)

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. of dry soil	
20-4-1936 original soil	0·00152	0·043	0·494	6·5	Nitric nitrogen varied from 0·00308 % to 0·0032 %
5-5-1936	0·00174	0·043	1·2266	17·5	
22-5-1936	0·00207	0·043	1·1654	29·2	
8-6-1936	0·0025	0·0454	1·0912	110·0	
24-6-1936	0·00294	0·0454	1·0091	175·0	
14-7-1936	0·00341	0·0466	0·9299	235·0	
26-7-1936	0·00368	0·0473	0·8516	292·6	
10-8-1936	0·00384	0·0477	0·7634	355·8	
12-10-1936	0·00341	0·048	0·5106	315·5	

Table III

1 kilogram of soil + 20 gms. Dextrin exposed (Temp. 34°—42°)

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. of dry soil	
8-10-1936 original soil	0·00145	0·057	0·6156	5·2	Nitric nitrogen varied from 0·0032% to 0·00376%
28-10-1936	0·00152	0 057	1·3564	6·1	
17-11-1936	0·0021	0·059	1·2772	12·6	
10-12-1936	0·0028	0·0608	1·1926	17·2	
28-12-1936	0·00364	0·062	1·0742	20·5	
18-1-1937	0·004	0·0636	0·9414	28·5	
4-2-1937	0·0038	0·0646	0·7728	20·5	
20-2-1937	0·00336	0·064	0·6292	18·5	
6-3-1937	0·00304	0·0636	0·6086	11·5	
28-3-1937	0·00258	0·0626	0·5984	8·5	

Table IV

1 kilogram of soil + 20 gms. Dextrin dark (Temp. 28°—31°)

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. of dry soil	
8-10-1936 original soil	0·00145	0·057	0·6156	5·2	Nitric nitrogen varied from 0·0032% to 0·0031%
28-10-1936	0·00145	0·057	1·3892	4·8	
17-11-1936	0·0016	0·057	1·3258	18·7	
10-12-1936	0·00188	0·0586	1·2644	32·5	
28-12-1936	0·0021	0·059	1·1916	82·5	
18-1-1937	0·00224	0·06	1·1032	150·5	
4-2-1937	0·00228	0·0604	0·9778	198·5	
20-2-1937	0·00224	0·0608	0·8454	225·0	
6-3-1937	0·00204	0·0612	0·6868	280·0	
28-3-1937	0·00196	0·0608	0·6648	205·0	

Table V

1 kilogram of soil + 20 gms. Maltose exposed (Temp. 34°-42°)

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. of dry soil	
8-10-1936 original soil	0·00145	0·057	0·6156	5·2	Nitric nitrogen varied from 0·0032 % to 0·00388 %
26-10-1936	0·0016	0·057	1·3684	4·2	
12-11-1936	0·00215	0·059	1·2672	16·2	
1-12-1936	0·0027	0·0604	1·1694	21·5	
17-12-1936	0·00322	0·0618	1·0712	32·8	
31-12-1936	0·004	0·0626	0·9658	30·2	
19-1-1937	0·0044	0·064	0·8334	31·5	
5-2-1937	0·00388	0·0656	0·6868	25·5	
22-2-1937	0·0032	0·064	0·6426	16·5	
7-3-1937	0·0028	0·0636	0·6284	12·5	
29-3-1937	0·0024	0·0626	0·6168	7·5	

Table VI

1 kilogram of soil + 20 gms. Maltose dark (Temp. 28°−31°)

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. of dry soil	
8-10-1936 original soil	0·00145	0·057	0·6156	5·2	Nitric nitrogen varied from 0·0032 % to 0·0031 %
26-10-1936	0·00148	0·057	1·3944	4·95	
12-11-1936	0·00168	0·057	1·3163	21·2	
1-12-1936	0·0019	0·0586	1·2451	40·5	
17-12-1936	0·0021	0·0596	1·1699	76·5	
31-12-1936	0·00256	0·0608	1·0873	125·6	
19-1-1937	0·0028	0·0612	0·9958	202·5	
5-2-1937	0·00284	0·0612	0·9062	215·0	
22-2-1937	0·0028	0·0612	0·7936	230·0	
7-3-1937	0·0024	0·0618	0·6652	250·0	
29-3-1937	0·00208	0·0612	0·6426	210·0	

Table VII

1 kilogram of soil + 20 gms. Fructose exposed (Temp. 34°–42°)

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. of dry soil.	
8-10-1936 original soil	0·00145	0·057	0·6156	5·2	Nitric nitrogen varied from 0·0032 % to 0·00372 %
25-10-1936	0·00168	0·057	1·3568	6·1	
11-11-1936	0·0021	0·0586	1·2622	13·2	
1-12-1936	0·0028	0·0608	1·1518	19·8	
17-12-1936	0·00336	0·0622	1·0264	36·0	
31-12-1936	0·00428	0·0636	0·8978	34·5	
19-1-1937	0·00452	0·0646	0·7614	29·8	
5-2-1937	0·0042	0·0656	0·6346	23·5	
22-2-1937	0·0038	0·0646	0·6284	15·0	
7-3-1937	0·00304	0·0636	0·6126	10·5	
29-3-1937	0·00271	0·0626	0·6028	7·0	

Table VIII

1 kilogram of soil + 20 gms. Fructose dark (Temp. 28°–31°)

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. of dry soil	
8-10-1936 original soil	0·00145	0·057	0·6156	5·2	Nitric nitrogen is 0·0032 %.
25-10-1936	0·00152	0·057	1·3745	7·2	
11-11-1936	0·00168	0·057	1·3161	19·8	
1-12-1936	0·00186	0·059	1·2429	35·8	
17-12-1936	0·00215	0·06	1·1633	82·5	
31-12-1936	0·00252	0·0608	1·0818	136·5	
19-1-1937	0·0028	0·0612	0·9876	225·5	
5-2-1937	0·00252	0·0618	0·8434	260·0	
22-2-1937	0·00248	0·0618	0·7318	275·0	
7-3-1937	0·0024	0·0622	0·6102	290·0	
29-3-1937	0·00224	0·0612	0·6028	210·0	

Table IX

1 kilogram of soil + 20 gms. Galactose exposed (Temp. 34° − 42°)

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. of dry soil	
19-10-1936 original soil	0·00145	0·057	0·6156	5·6	Nitric nitrogen varied from 0·0032 % to 0·004 %
10-11-1936	0·00176	0·057	1·3432	9·2	
26-11-1936	0·00224	0·06	1·2386	17·8	
10-12-1936	0·00289	0·0613	1·1128	19·2	
28-12-1936	0·00336	0·062	0·9802	22·5	
18-1-1937	0·004	0·064	0·8468	24·2	
4-2-1937	0·00388	0·065	0·7036	22·5	
20-2-1937	0·00336	0·066	0·5986	18·5	
6-3-37	0·0027	0·065	0·5628	10·5	
28-3-1937	0·0024	0·0636	0·5584	8·0	

Table X

1 kilogram of soil + 20 gms. Galactose dark (28° − 31°)

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. dry soil	
19-10-1936 original soil	0·00145	0·057	0·6156	5·6	Nitric nitrogen varied from 0·0032 % to 0·00336 %
10-11-1936	0·00152	0·057	1·3618	13·6	
26-11-1936	0·00172	0·057	1·2904	26·8	
10-12-1936	0·0019	0·059	1·2092	38·6	
28-12-1936	0·0021	0·06	1·1176	92·8	
18-1-1937	0·00252	0·0604	1·0178	195·5	
4-2-1937	0·00256	0·0608	0·8942	225·0	
20-2-1937	0·00268	0·0618	0·7456	295·0	
6-3-1937	0·0024	0·0618	0·6492	245·0	
28-3-1937	0·0021	0·0612	0·6284	215·0	

Table XI

Plot 4′ × 4′ containing 2 kilograms of Glycerol

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. of dry soil	
12-10-1936 original soil	0·0009	0·042	0·4603	1·95	Nitric nitrogen varied from 0·002 % to 0·003 %
30-10-1936	0·0012	0·042	0·7973	2·8	
18-11-1936	0·00186	0·0437	0·7317	9·5	
5-12-1936	0·0021	0·0451	0·6702	20·6	
25-12-1936	0·0028	0·0466	0·5878	23·5	
18-1-1937	0·00197	0·0472	0·4954	17·5	
3-2-1937	0·0019	0·0472	0·4528	15·0	
23-2-1937	0·0014	0·0466	0·4468	10·0	
22-3-1937	0·00134	0·0461	0·4356	6·5	

Table XII

Plot 4′ × 4′ containing 2 kilograms of Starch

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. of soil	
12-10-1936 original soil	0·00087	0·0466	0·5033	2·2	Nitric nitrogen varied from 0·00172 % to 0·0024 %
30-10-1936	0·00124	0·0466	0·9213	3·1	
18-11-1936	0·00174	0·0482	0·8731	8·8	
5-12-1936	0·002	0·05	0·8205	19·8	
25-12-1936	0·00268	0·0512	0·7442	20·5	
18-1-1937	0·002	0·0525	0·6456	16·5	
3-2-1937	0·00197	0·0531	0·5604	14·5	
23-2-1937	0·00174	0·0525	0·5518	9·5	
22-3-1937	0·0014	0·0518	0·5384	5·5	

Table XIII

Control Plot $4' \times 4'$

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. of soil	
12-10-1936	0·00116	0·054	0·567	2·47	Nitric nitrogen 0·002 % is constant
30-10-1936	0·00116	0·054	0·567	2·52	
18-11-1936	0·00116	0·054	0·567	2·01	
5-12-1936	0·00116	0·054	0·567	2·92	
25-12-1936	0·00116	0·054	0·567	2·22	
18-1-1937	0·00116	0·0538	0·5328	1·95	
3-2-1937	0·00116	0·0538	0·5286	2·5	
23-2-1937	0·00105	0·0538	0·5286	2·05	
22-3-1937	0·00105	0·0538	0·5248	2·5	

Discussion

The foregoing results show that although in the dark the Azotobacter numbers are approximately ten times than in the dishes exposed to sunlight, the nitrogen fixation in dishes receiving sunlight is almost double than in those kept in dark. Moreover a far-reaching conclusion can be drawn from the above experimental results when the amount of nitrogen fixed per gram of carbon oxidised with various energy-rich materials is calculated. The following table shows the amount of nitrogen fixed per gram of carbon oxidised both in light and dark.

Table XIV

Substance	Nitrogen fixed	
	Light	Dark
Mannitol	12·82 mgm.	6·98 mgm.
Dextrin	13·0	5·98
Maltose	12·6	6·58
Fructose	11·91	6·8
Galactose	12·08	6·73

The above results show conclusively that for the same amount of carbon oxidised of the energy materials the nitrogen fixation in light is practically double than that in the dark. Dhar and Mukerji[1] have explained that the formation of

nitric oxide may be the first step in the fixation of nitrogen. It is well known that the combination of nitrogen and oxygen forming nitric oxide according to the equation

$$N_2 + O_2 + 43{\cdot}2 \text{ Cal} \rightarrow 2 NO$$

is an endothermal chemical change which should take place when supplied with required energy. When the energy materials are mixed with soil they undergo oxidation as is evident from the decrease of carbon with time. A gram molecular weight of these energy materials in their oxidation liberate energy much in excess of that required for the combination of nitrogen and oxygen forming nitric oxide. Even in the dark nitrogen fixation takes place in soils when supplied with energy materials but the efficiency of this process is very low. When, however, the soils containing the energy materials are exposed to light daily for eight hours, not only the velocity of oxidation of the energy materials will increase as is evident from the quicker disappearance of carbon in light than in dark but the efficiency of the process is also considerably increased. It seems therefore that the nitrogen fixation which is an endothermal reaction, is considerably aided not only by the energy obtained from the oxidation of the carbohydrates but also by the absorption of the incident light. As there is an intimate contact between nitrogen and oxygen on the soil surface, which is receiving energy both from the sun and from the oxidation processes, it is quite reasonable to expect more nitrogen fixation in light than in dark and this is actually observed experimentally. Hence light actually takes part in the nitrogen fixation in the soils when supplied with the energy materials.

In the field trials with starch and glycerol I find the same behaviour as in dishes, the carbon decreases and the nitrogen increases. Hence in fields when the energy materials like glycerol and starch undergo oxidation, nitrogen is fixed in the soil. No change either in the total nitrogen or Azotobacter numbers is observed in the control plot.

My thanks are due to the Imperial Council of Agricultural Research for a grant which enabled me to do the above work.

I am also indebted to Prof. N. R. Dhar, D.Sc., for his kind suggestion and keen interest in this work.

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CHANGES IN SOIL NITROGEN AFTER THE ADDITION OF FRESH COWDUNG TO SOIL

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Communicated by Prof. N. R. Dhar

Received February 26, 1937

SUMMARY

1. Fresh cowdung has been shown to fix atmospheric nitrogen in dishes, in fields and in sterile vessels exposed to sunlight.

2. The amount of nitrogen fixed as well as the amount of nitrogen fixed per gram of carbon oxidised is greater in light than in the dark.

3. Molasses seems to stimulate the oxidation of cowdung in the soil, and increases the total amount of nitrogen fixed.

4. Under the conditions of these experiments no white ants were seen to develope in the soil after the treatment with cowdung.

The following lines from Waksman's *Principles of Soil Microbiology* 1932, pp. 491, show that different results have been obtained by different workers regarding the fixation of nitrogen on the addition of cellulosic matters to the soil, "Polysaccharides, like cellulose, can also serve as valuable sources of energy if they are first partially broken down by cellulosic decomposing organisms [1,5,6,7,8]. These results need further confirmation; this process is of particular importance, since the bulk of the energy material commonly added to the soil consists of cellulose and pentosans" and incontestable proof of nitrogen fixation in the soil is still lacking. There are indications in the literature that certain hemicelluloses, including pentosans, can be utilised as direct source of energy in nitrogen fixation.

The object of the present investigation is to find out whether cowdung which consists mainly of pentosans and cellulosic materials together with a small percentage of protein nitrogen, when added to the soil can bring about nitrogen fixation in the soil.

From time immemorial cowdung is used as a fertiliser in the agricultural practice in India and in other parts of the world. In India the farmers generally store the animal dung in pits near their houses and transfer to the fields two to three months before the sowing season.

Various methods of composting have been put forward to make cowdung suitable for crops. The general belief is that fresh cowdung will produce harmful effect on the crops. The main principle of all these processes is to get rid of most of the carbonaceous matters, which are thought to exert a bad effect on plant growth. Thus only a part of the total nitrogen originally present in the cowdung together with its whole mineral content is transferred to the fields. The whole of the easily oxidisable carbonaceous substances and some cellulosic matters are destroyed by anaerobic micro-organisms and the residue contains less organic matter than before.

In recent publications from this laboratory [2,3] it has been shown that energy-rich materials when added to soil stimulate nitrogen fixing factors and considerable amount of nitrogen fixation takes place in the soil. Therefore if cowdung is added directly to soil a part of the energy derived from the oxidation of the organic materials present in cowdung may be utilised to enrich the soil nitrogen, besides other ingredients already present in cowdung.

The following are the carbon and nitrogen content of a few samples of fresh cowdung :—

Table I

No.	NITROGEN	CARBON	C/N	No.	CARBON	NITROGEN	C/N
1.	0·3684%	7·499%	20·3	4.	8·194%	0·3189%	25·7
2.	0·358	9·487	27·5	5.	11·68	0·3783	32·3
3.	0·2626	12·409	21·5	6.	6·841	0·378.	18·1

These results show that the carbon-nitrogen ratio in cowdung is greater than 20 and nitrogen is low; therefore large amounts of dung manure is generally added to the fields for a high crop yield. Moreover in the method of composting the organic carbon is practically destroyed and the finished product containing 1 to 2 per cent nitrogen when applied to the fields is attended by losses of nitrogen in gaseous form.[4]

The loss of nitrogen by this process may amount to two to three times that taken up by a crop. Therefore we must mould our agricultural practices in such a way as to utilise the whole of the nitrogen added, prevent the nitrogen loss and stimulate nitrogen fixing factors in the soil. This problem of nitrogen changes in the soil after an application of cowdung needs a thorough scientific investigation for a clear elucidation of the chemical processes, which are of high economic and scientific interest.

Accordingly, experiments were undertaken to study the nitrogen changes which take place in soil and in dishes after treating with cowdung. The effect of sunlight on these reactions has also to be investigated. The following are the experimental results obtained :—

Table II

Soil=500 gms.+cowdung=100 gms. Started on 28-7-1936.

Condition.	NH_3-N,	NO_3-N,	Total−N,	Total−C.	Date of Sampling
Continued from Presidential Address, National Acad. Sci. (India) 1937, pp. 30.					
Light.	0·002%	0·0036%	0·1308%	1·1068%	12-1-1937
Shade.	0·001	0·0022	0·1288	1·0467	12-1-1937
Light.	0·0024	0·0042	0·0986	0·9244	24-4-1937
Shade.	0·0012	0·0018	0·1186	1·00	24-4-1937

Table III

Soil=500 gms.+cowdung=100 gms.+molasses 5 gms. Started on 28-7-1936.

Continued from Presidential Address, National Acad. Sci. (India) 1937, pp. 30.					
Light.	0·0024	0·003	0·1378	1·1161	12-1-1937
Shade.	0·0017	0·0022	0·1288	0·9542	12-1-1937
Light.	0·0018	0·0037	0·1012	1·042	24-4-1937
Shade.	0·0014	0·0018	0·1082	0·8642	24-4-1937

These results show that nitrogen fixation takes place when cowdung is mixed with soil and is kept under natural conditions. These observations were expected to hold good under field conditions. Field experiments were performed to verify these observations under field conditions. The following are the results obtained.

Table IV

Plot area=36 sq. ft. Cowdung 10 kg. Started on 20-11-1936.

NH_3-N,	NO_3-N,	Total−N,	Total-C	Date of Sampling.
0·0017%	0·002%	0·0583%	0·6899%	20-11-1936
0 0017	0·002	0·0626	0·5859	23-12-1936
0·0014	0·0024	0·0631	0·5224	19-1-1937
0·0012	0·0025	0·0667	0·4286	19-2-1937
0·0013	0·0026	0·0609	0·4312	17-3-1937
0·0014	0·0029	0·0544	0·4312	4-5-1937

Table V

Plot area=36 sq. ft. Cowdung=30 kg.+Molasses=30 kg. Started on 20-11-1936.

0·0021	0·0028	0·09	2·184	20-11-1936
0·0043	0·0028	0·0934	1·505	23-12-1936
0·002	0·0028	0·10	1·138	19-1-1937
0·0022	0·0028	0·1024	0·8214	19-2-1937
0·0015	0·0030	0·0823	0·8285	17-3-1937
0·0014	0·0032	0·0812	0·8124	4-5-1937

Table VI

Plot area=36 sq. ft. Cowdung 60 kg.+Molasses 60 kg. Started on 20-11-1936.

0·004	0·0046	0·14	4·035	20-11-1936
0·0023	0·0046	0·14	3·753	23-12-1936
0·0028	0·0031	0·14	2·511	19-1-1937
0·0030	0·0031	0·129	1·3424	19-2-1937
0·002	0·0031	0·1273	0·8614	17-3-1937
0·0013	0·0037	0·1013	0·8444	4-5-1937

In recent publications (*loc. cit.*) sunlight has been shown to play an important part in the soil processes. Experiments were undertaken to verify these conclusions under field conditions. On this account similarly treated fields were covered with wooden planks to avoid direct exposure to sunlight and some were exposed to sunlight. No watering was done and the plots were ploughed one evening in each month. The following are the experimental results.

Table VII

Plot area=36 sq. ft. Cowdung=30 kg. Exposed to sunlight.

NH_3-N,	NO_3-N,	Total−N,	Total−C,	Date of Sampling.
0·0018%	0·0018%	0·0636%	0·8801%	20-11-1936
0·0015	0·002	0·0742	0·6921	19-1-1937
0·0013	0·0022	0·0751	0·5212	19-2-1937
0·0012	0·0024	0·0714	0·555	17-3-1937
0·0021	0·0031	0·0651	0·535	4-5-1937

Table VIII

Covered with wooden planks.

0·0015	0·0018	0·0584	0·8453	20-11-1936
0·0009	0·0018	0·0583	0·644	19-1-1937
0·0008	0·0019	0·0592	0·5821	19-2-1937
0·0008	0·0018	0·0538	0·6163	17-3-1937
0·0006	0·0014	0·0533	0·6012	4-5-1937

Table IX

Plot area=36 sq. ft. Cowdung 19·5 kg. Exposed to sunlight.

0·0017	0·0025	0·0692	0·8616	20-11-1936
0·0014	0·0028	0·0768	0·6112	19-1-1937
0·0012	0·0028	0·0774	0·5001	19-2-1937
0·0008	0·0029	0·0636	0·5306	17-3-1937
0·0014	0·0038	0·0682	0·534	4-5-1937

Table X

Covered with wooden planks.

0·0017	0·0026	0·07	0·8692	20-11-1936
0·0011	0·0018	0·0731	0·6538	19-1-1937
0·0011	0·0018	0·0738	0·5112	19-2-1937
0·0007	0·0018	0·07	0·5692	17-3-1937
0·0005	0·0012	0·0694	0·5342	4-5-1937

Table XI

Plot area = 36 sq. ft. Cowdung 60 kg. Exposed to sunlight.

NH_3-N,	NO_3-N,	Total−N,	Total−C,	Date of Sampling.
0·0018 %	0·0018 %	0·0744 %	0·9703 %	20-11-1936
0·0017	0·0025	0·0832	0·8212	19-1-1937
0·0014	0·0026	0·0841	0·6112	19-2-1937
0·0015	0·0028	0·0778	0·6812	17-3-1937
0·0028	0·0031	0·0712	0·6424	4-5-1937

Table XII

Covered with wooden planks

0·0025	0·0031	0·0875	1·672	20-11-1936
0·0013	0·0022	0·0862	0·937	19-1-1937
0·001	0·0018	0·0854	0·8114	19-2-1937
0·0009	0·0018	0·0812	0·6277	17-3-1937
0·0007	0·0011	0·08	0·6172	4-5-1937

These results indicate that cowdung along with the cellulosic materials when turned into soil under Indian conditions is more beneficial than that of composting. The treated fields show a greater percentage of nitrogen which goes on increasing till most of the organic matter is oxidised and then it slightly decreases due to the loss of nitrogen in the process of nitrification. The evolution of carbondioxide which is enhanced by the addition of organic matter brings some of the calcium, iron, phosphates and magnesium carbonates into solution in the form of mostly bi-carbonates. These soluble salts enhance the microbiological and oxidation processes in the soil.

Another important point has been brought out in this work is that the process of fixation of atmospheric nitrogen apart from the well-known bacterial action is also partly photochemical, catalytic and thermal process. The fields covered with wooden planks show less fixation per gram of carbon oxidised than those plots receiving direct sunlight. Similar results have been obtained with soils in dishes.

To study the oxidation and fixation of atmospheric nitrogen by cowdung under perfectly sterilised conditions the following arrangements were made. Pure air completely freed from ammonia, nitric oxide, carbondioxide and bacteria by passing through solutions of ferrous sulphate and sulphuric acid, concentrated sulphuric acid and baryta is bubbled through a sterile suspension of cowdung and water in a quartz flask. The air then passes through a solution of standard baryta to catch the carbondioxide and then another flask containing dilute sulphuric acid to catch ammonia. Two more flasks containing baryta and dilute sulphuric acid are joined in series to prevent the incidence of any ammonia or carbondioxide from outside.

A similar experiment was performed under identical conditions except that the reaction vessel was that of ordinary glass and wrapped thoroughly by a black cloth to cut off light. Sterilisation of the suspension, container and the connections were done in an autoclave at 15 lbs. pressure for three hours. To test for complete sterilisation a control emulsion was similarly treated at the same time and plated for Azotobacter and total bacteria. No colony was seen to develop even after ten days' incubation. The following are the results obtained after 100 hours' exposures to sunlight.

Original carbon in the mixture of soil and cowdung		0·9372 %
„ Nitrogen „ „		0·05935 %
Total nitrogen left in the exposed vessel	(*a*)	0·06034 %
	(*b*)	0·06087
	(*c*)	0·06034
	(*d*)	0·05932
Total carbon in the exposed vessel	(*a*)	0·837 %
	(*b*)	0·8375
	(*c*)	0·837
	(*d*)	0·837

Total nitrogen in the vessel kept in the sunlight wrapped in a black cloth.	(*a*)	0·05931%
	(*b*)	0·05908
Total carbon left in the vessel	(*a*)	0 9207
	(*b*)	0·9207

Mean carbon oxidised in the light :—0·1002 gm.
Mean nitrogen fixed in the light :—0·86 ml. gm.
Mean carbon oxidised in the dark :—0·0165 gm.
Mean nitrogen fixed in the dark :—nil

This as well as the dish and the field experiments demonstrate clearly that not only the amount of nitrogen fixation is greater in sunlight, but the amount of

nitrogen fixed per gram of carbon oxidised is also greater in light. Hence it is assumed that sunlight takes a direct part in the process of nitrogen fixation from the energy derived from the oxidation of the energy-rich materials in soil.

Thanks are due to Prof. N. R. Dhar, D.Sc., I.E.S., etc., for his kind guidance and interest in this work.

N.B.—As the publication of the paper was delayed for a long time a few minor additions had to be made.

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PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES
INDIA

Part 2] May, 1937 [Volume 7

THE MATHEMATICAL THEORY OF A NEW RELATIVITY

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Received July 8, 1937.

CHAPTER XV

The Rotational Theory of Light and Matter

SECTION I

Introduction

I. "There is something radically wrong with the present fundamental conceptions of Physics, and we do not see how to set it right." (Eddington)[1] Light and matter show at the same time both the particle and the wave aspects—an apparent contradiction. As the true picture can not be visualised, it has in despair been completely discarded. Physics is now dominated by purely artificial mathematical devices, which seem to work well, without disclosing their real Physical significance. When one tries to form a mental conception, there is nothing but confusion and chaos. Everything consists of waves, stretching from infinity to infinity; yet they are not real waves, not even waves of energy, but a mere mathematical fiction. They are "waves of probability or chance." Jeans would regard these waves as consisting of "knowledge". Max Born thinks "it would be better to call them waves of partial knowledge". Unkind critics call them "waves of ignorance". As matters stand, all such descriptions

are equally appropriate. Wave Mechanics evades the questions how energy is propagated through space without matter or medium, why the waves do not scatter away or how an infinite train of waves can suddenly converge to a point when meeting another such train ; nor has it yet at all tackled numerous other problems of Physics including polarisation. De Broglie wave associated with the electron remains mysterious and obscure. "Unfortunately, however, it (Wave Mechanics) compels one to use a continuum the number of whose dimensions is not that ascribed to Physics hitherto (four) but rises indefinitely with the number of particles constituting the system under consideration" (Einstein),[2] being three times as many. It is an irony of fate that Relativity in order to explain the interaction of a million stars should require a space of four million dimensions. Wave Mechanics using imaginary operators and abounding in $\sqrt{-1}$ causes bewilderment.

2. I have already ventured to put forward a Rotational Theory of Light essentially corpuscular in character. Light consists of swarms of discreet units, Newton's corpuscles, now called radions, with one addition that in its inner structure a radion consists of two components of equal and opposite charges (found to be of equal masses also) rotating round their common centre of gravity, which is moving forward with a uniform velocity. The translational and the rotational motions give it both the particle and wave aspects simultaneously e. g. both momentum and wavelength. Light then becomes an electromagnetic system, creating and carrying its field with it. Maxwell's equations follow naturally. It has been shown that such a system automatically propagates itself with the known velocity of light.

3. Further developments are offered in this Chapter. The orbit, referred to the moving axes, may not be perpendicular but may be inclined to the path of motion. A hydrogen atom, consisting of a positively charged proton and a negatively charged electron, has a similar structure. Rotational mechanics applies to it in a similar way, only the masses are unequal and the orbit is generally elliptical. The law u. $v = c^2$ follows from this rotational cum translational motion. De Broglie's assumption p. $\lambda = h$ is then deduced. Einstien's formula $E = mc^2$ is explained. Planck's assumption $E = h\nu$ is also deduced. Heisenberg – Schrodinger's equation

$$\left[\nabla^2 + \frac{8\pi^2 m}{h^2}(E - V)\right]\psi = 0$$

is easily obtained, with other necessary results. Lorentz transformations utilised in Relativity are explained. Thus many mysteries are solved, as the picture is clearly before the mind. Some further developments, reserved for a later paper, will be interesting.

4. For the solution of Maxwell's equations when the E's are expressed in exponentials I express my indebtedness to Dr. N. S. Japolsky, whose method and results are acknowledged in Section III headed as "Japolsky's Solutions". For light, I have added a simpler Physical solution.

There is undoubtedly some resemblance between his rotating electromagnetic waves or "whirls" stretching from infinity to infinity, or the whole train either bodily moving along the axis of symmetry or rotating round it, and my rotatary binary system moving forward from the source to the obstacle, carrying its own electromagnetic field with it. There are many common features between the two, and part of the mathematics certainly seems to be common too. But there are also a few essential differences:—

(1) The Physical nature of the whirls is not known; while radions are like binary stars, revolving and moving forward.

(2) A rotating wave or whirl requires a medium for its propagation, and a fluid medium too; it is awkward to fill the whole space with fluid medium in order to find means for light to come from any single star. A radion can travel by itself through the void space; indeed a medium would obstruct it.

(3) A whirl may well expand and spread away; so some assumption has to be made for its retaining form and size during its passage through vast space. On the other hand, no such assumption as to the stability of a binary system is required, for the mutual attraction of the components will maintain their orbit.

(4) A whirl should create more and more whirls round itself in the medium and its energy would soon be scattered away. An isolated radion conserves its energy; until it meets an obstruction, it gets no chance for parting with any energy.

(5) It is not clear why a whirl of proton or electron should possess a characteristic mass. Another difficulty is that such energy or mass would have to be distributed all along its infinite length and singularities should not occur. No such difficulty arises in the case of discreet radionic units.

(6) It is difficult to see what happens when two whirls collide. If they are superposed, they should pass through. If not, then after collision the two infinite trains must scatter away ; but it is difficult to conceive of a Compton scattering of two infinite waves approaching each other from opposite directions along the same axis. Radions can of course collide like billiard balls.

(7) It is doubtful whether without any charged particles forces acting on the field can at all be produced.[3] The charged binary system will of course produce a field.

But these difficulties are not peculiar to Japolsky's "whirls". They are inherent in every system of waves, including those of Wave Mechanics.

5. I again express my deep gratitude to Dr. D. S. Kothari, M. Sc., Ph. D., Reader in Physics at the Delhi University and Mr. Ram Niwas Rai, M. Sc., Lecturer in Physics at the Allahabad University for their great kindness in making several valuable suggestions.

SECTION II

Electromagnetic Equations

Let E and M denote electric and magnetic intensities and suffixes indicate their components. Then Maxwell's electromagnetic equations in cylindrical coordinates, as given in my Rotational Theory of Light[4] for a region free from charges are :—

$$\left.\begin{aligned} \frac{k}{c}\frac{\partial E_r}{\partial t} &= \frac{1}{r}\frac{\partial M_z}{\partial \theta} - \frac{\partial M_\theta}{\partial z} \\ \frac{k}{c}\frac{\partial E_\theta}{\partial t} &= \frac{\partial M_r}{\partial z} - \frac{\partial M_z}{\partial r} \\ \frac{k}{c}\frac{\partial E_z}{\partial t} &= \frac{1}{r}\frac{\partial (rM_\theta)}{\partial r} - \frac{1}{r}\frac{\partial M_r}{\partial \theta} \end{aligned}\right\} \quad \left.\begin{aligned} -\frac{\mu}{c}\frac{\partial M_r}{\partial t} &= \frac{1}{r}\frac{\partial E_z}{\partial \theta} - \frac{\partial E_\theta}{\partial z} \\ -\frac{\mu}{c}\frac{\partial M_\theta}{dt} &= \frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} \\ -\frac{\mu}{c}\frac{\partial M_z}{\partial t} &= \frac{1}{r}\frac{\partial (rE_\theta)}{\partial r} - \frac{1}{r}\frac{\partial E_r}{\partial \theta} \end{aligned}\right\}$$

$$\left.\begin{aligned} \frac{1}{r}\frac{\partial}{\partial r}(rE_r) + \frac{1}{r}\frac{\partial E_\theta}{\partial \theta} + \frac{\partial E_r}{\partial z} &= 0 \\ \frac{1}{r}\frac{\partial}{\partial r}(rM_r) + \frac{1}{r}\frac{\partial M_\theta}{\partial \theta} + \frac{\partial M_z}{\partial z} &= 0 \end{aligned}\right\} \quad \ldots \qquad (43{\cdot}1)$$

Proceeding as in Ch. XIV Sec. V, we differentiate the first group with respect to t, eliminate the magnetic components by substitution from the second group, and get

$$\left.\begin{aligned}
\frac{\mu k}{c^2}\frac{\partial^2 E_r}{\partial t^2} &= -\frac{1}{r^2}\frac{\partial^2 (rE\theta)}{\partial r\partial\theta} + \frac{1}{r^2}\frac{\partial^2 E_r}{\partial\theta^2} + \frac{\partial^2 E_z}{\partial z^2} - \frac{\partial^2 E_z}{\partial r\partial z}\\
\frac{\mu k}{c^2}\frac{\partial^2 E\theta}{\partial t^2} &= -\frac{1}{r}\frac{\partial^2 E_z}{\partial z\partial\theta} + \frac{\partial^2 E\theta}{\partial z^2} + \frac{\partial}{\partial r}\left(\frac{\partial (r\,E\theta)}{r\partial r}\right) - \frac{\partial}{\partial r}\left(\frac{\partial E_r}{r\partial\theta}\right)\\
\frac{\mu k}{c^2}\frac{\partial^2 E_z}{\partial t^2} &= -\frac{\partial}{r\partial r}\left(r\frac{\partial E_r}{\partial z}\right) + \frac{\partial}{r\partial r}\left(r\frac{\partial E_z}{\partial r}\right) + \frac{\partial^2 E_z}{r^2\partial\theta^2} - \frac{\partial^2 E\theta}{r\partial\theta dz}\\
\text{Also}\quad \frac{1}{r}\frac{\partial}{\partial r}(r\,E_r) &+ \frac{1}{r}\frac{\partial E\theta}{\partial\theta} + \frac{\partial E_z}{\partial z} = 0
\end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (43{\cdot}2)$$

Similarly we obtain,

$$\left.\begin{aligned}
\frac{\mu k}{c^2}\frac{\partial^2 M_r}{\partial t^2} &= -\frac{1}{r^2}\frac{\partial^2 (r\,M\theta)}{\partial r\partial\theta} + \frac{1}{r^2}\frac{\partial^2 M_r}{\partial\theta^2} + \frac{\partial^2 M_r}{\partial z^2} - \frac{\partial^2 M_z}{\partial r\partial z}\\
\frac{\mu k}{c^2}\frac{\partial^2 M\theta}{\partial t^2} &= -\frac{1}{r}\frac{\partial^2 M_z}{\partial z\partial\theta} + \frac{\partial^2 M\theta}{\partial z^2} + \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial (rM\theta)}{\partial r}\right] - \frac{\partial}{\partial r}\left(\frac{\partial M_r}{r\partial\theta}\right)\\
\frac{\mu k}{c^2}\frac{\partial^2 M_z}{\partial t^2} &= -\frac{\partial}{r\partial r}\left(r\frac{\partial M_r}{\partial z}\right) + \frac{\partial}{r\partial r}\left(r\frac{\partial M_z}{\partial r}\right) + \frac{\partial^2 M_z}{r^2\partial\theta^2} - \frac{\partial^2 M\theta}{r\partial\theta\partial z}\\
\frac{1}{r}\frac{\partial}{\partial r}(r\,M_r) &+ \frac{1}{r}\frac{\partial M\theta}{\partial\theta} + \frac{\partial M_z}{\partial z} = 0
\end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (43{\cdot}3)$$

SECTION III

Japolsky's Solutions

1. N. S. Japolsky[5] has considered the case where the components of the electromagnetic field are simple harmonic functions of t, z and θ and can be expressed as $E.\, e^{i(wt + \kappa z + n\theta)}$ where the coefficients may be complex. He has in a most elegant manner obtained the solutions of Maxwell's equations by transforming them into Bessel's equations. For the purposes of this paper it will be quite sufficient to find the expression for E_z. The others will be merely reproduced.

Let $E_r = R e^{i\phi}$; $E_\theta = \theta e^{i\phi}$; $E_z = Z e^{i\phi}$,

where R, θ, Z are functions of r only and $\phi = wt + \kappa z + n\theta$. Substituting these values in the first three equations of (43·2), and putting

$c_1 = \frac{c}{\sqrt{\mu k}}$ we get

$$-\frac{w^2}{c_1^2}R = -\frac{in}{r^2}\frac{\partial(r\theta)}{\partial r} - \frac{n^2}{r^2}R - \kappa^2 R - i\kappa\frac{\partial Z}{\partial r}.$$

$$-\frac{w^2}{c_1^2}\theta = \frac{n\kappa}{r}Z - \kappa^2\theta + \frac{\partial}{\partial r}\left[\frac{\partial(r\theta)}{r\partial r}\right] - in\frac{\partial}{\partial r}\left(\frac{R}{r}\right)$$

$$-\frac{w^2}{c_1^2}Z = -i\kappa\frac{\partial}{r\partial r}(rR) + \frac{\partial}{r\partial r}\left(r\frac{\partial Z}{\partial r}\right) - \frac{n^2}{r^2}Z + \frac{n\kappa}{r}\theta. \quad \ldots \quad (44·1)$$

We multiply the first equation by $-ir$ and then differentiate with respect to r. We multiply the second equation by n. We multiply the third equation by κ r. If we then add up the three results we find that all the terms on the right hand side cancel one another. Therefore omitting the common factor $-\frac{w^2}{c_1^2}$, we have,

$$\kappa r Z - i\frac{\partial}{\partial r}(rR) + n\theta = 0 \quad \ldots \quad \ldots \quad (44·2)$$

We can now eliminate R and θ by multiplying (44·2) by $\frac{\kappa}{r}$ and adding it to the last equation of (44·1), and get,

$$\frac{\partial}{r\partial r}\left(r\frac{\partial Z}{\partial r}\right) + \left(\frac{w^2}{c_1^2} - \kappa^2 - \frac{n^2}{r^2}\right)Z = 0 \quad \ldots \quad \ldots \quad (44·3)$$

Substituting $\rho = \sqrt{\frac{w^2}{c_1^2} - \kappa^2}\,.\,r$ and working out the differentiation of the first term of (44·3) and also substituting total differentials because R, θ, Z are functions of r only we get,

$$\left[\frac{d^2}{d\rho^2} + \frac{d}{\rho d\rho} + \left(1 - \frac{n^2}{\rho^2}\right)\right]Z = 0 \quad \ldots \quad \ldots \quad (44·4)$$

The above equation is a well-known Bessel equation whose solution that is finite throughout the space, is

$$Z = A\, J_n(\rho)\ , \text{ where } \rho = \sqrt{\frac{w^2}{c_1^2} - \kappa^2}.\, r,$$

and $J_n(\rho)$ is a Bessel coefficient of the order n of the variable ρ ... (44·5) For real waves, the independent variable ρ should be real otherwise the amplitude will tend to ∞ when r approaches ∞.

He has by analogous transformations also obtained the expressions for the other quantities.

$$\left.\begin{aligned} i\,R &= -A_1 J_{n+1} + A_2 J_{n-1} \\ \theta &= A_1 J_{n+1} + A_2 J_{n-1} \end{aligned}\right\} \quad \ldots \qquad \ldots \quad (44\cdot6)$$

where however $A = -\sqrt{\frac{w^2}{c_1^2} - \kappa^2}.\ \frac{A_1 + A_2}{\kappa}$.

Also if the magnetic forces be $M_r = R'.\, e^{\,i\,(wt + \kappa z + n\theta)}$ etc,

then

$$\left.\begin{aligned} Z' &= i\sqrt{\frac{w^2}{c_1^2} - \kappa^2}.\,(A'_1 + A'_2)\, J_n \\ R' &= -A_1' J_{n+1} + A'_2 J_{n-1} \\ \theta' &= i\,(A_1' J_{n+1} + A'_2 J_{n-1} \end{aligned}\right\} \quad \ldots \quad (44\cdot7)$$

where

$$\left.\begin{aligned} A'_1 &= -\frac{c\kappa}{w}\left[A_1 + \tfrac{1}{2}(A_1 + A_2)\left(\frac{w^2}{\kappa^2 c_1^2} - 1\right)\right] \\ A_2' &= +\frac{c\kappa}{w}\left[A_2 + \tfrac{1}{2}(A_1 + A_2)\left(\frac{w^2}{\kappa^2 c_1^2} - 1\right)\right] \end{aligned}\right\} \quad (44\cdot8)$$

2. Japolsky [6] has taken these equations as meaning a system of rotating electromagnetic waves called "whirls" stretching from $-\infty$ to $+\infty$.

By assuming that they are real, and that they do not scatter away (as waves in a medium should) but somehow remain stable and even retain their shape, he has been able to deduce the most remarkable result

$$c_2.\, v_z = c^2 \quad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \quad (44\cdot9)$$

where $c_2 = -\frac{w}{\kappa}$, the phase velocity, and v_z is the velocity with which the whole wave train with its central plane moves in the axial direction.

He has obtained de Broglie's formula in the form

$p.\lambda = \frac{h_1}{q}\left(1 + \sqrt{1 - \frac{v^2}{c^2}}\right)$, which would make the product vary with v.

He also appears to have obtained certain other important results including Relativity relations in a world of whirls. His general Theory of Relativity appears to be different as it depends on the actual structure of the system of whirls to which it is applied. I have not had the opportunity to check these.

SECTION IV

The Rotational Theory of Light.

A SIMPLER PHYSICAL METHOD.

1. In the Rotational theory of light there is a complete picture of a binary system, with its two components of equal and opposite charges rotating round their common centre of gravity, which is moving forward along the helical axis of symmetry with a uniform velocity. When the two components, the poson and the negon, of the light radion have equal masses, they must describe the same circular orbit. In such a case there can be no question of the rotation of the orbit and the force components must be independent of θ itself. Also as the two opposite charges always remain at the ends of a diameter, the phase difference between the radial and the transverse components of the force is $\frac{\pi}{2}$. Their orbit may have any inclination to the sectional plane at right angles to the axis of symmetry and moving with the system i. e. it may not be perpendicular to the Z−axis.

In Ch. XIV. Sec V, it was as a first approximation tacitly assumed that the forces acted instantaneously, and that in a circular orbit the transverse component $Q = 0$. Corrections for the rotating charged components were postponed (see para 4 p. 355). Now it is *not* assumed that $E_\theta = 0$. Similarly we may consider that the value of E_z is not known to be zero, though it may turn out to be so.

We first consider the motion with respect to fixed axes and origin at the centre. As after a complete period of revolution, the same conditions revert at every point, it is obvious that the force components must be periodic in time t, and z, but independent of θ. Also as the forces must depend on the distance of the point, which is ordinarily very large compared to the diameter of the binary orbit, they must be functions of r. Hence we can put

$$\left.\begin{aligned} E_r &= R.\cos\phi \\ E_\theta &= \theta.\sin\phi \\ E_z &= Z.\cos(\phi+\gamma). \end{aligned}\right\} \quad \ldots \quad \ldots \quad (45{\cdot}1)$$

where $\phi = wt + \kappa z$, w and κ being some unknown constants and R, θ, Z are functions of r only and γ denotes the phase difference of Z.

2. Substituting these values in the equations (43·2) and putting $c_1 = \dfrac{c}{\sqrt{\mu k}}$, we get

$$\left.\begin{aligned} -\frac{w^2}{c_1^2}.R\cos\phi &= -\kappa^2.R.\cos\phi \\ &\quad + \kappa\frac{\partial Z}{\partial r}(\sin\phi.\cos\gamma + \cos\phi.\sin\gamma) \\ -\frac{w^2}{c_1}\theta.\sin\phi &= -\kappa^2.\theta.\sin\phi + \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial(r\theta)}{\partial r}\right]\sin\phi \\ -\frac{w^2}{c_1^2}.Z.(\cos\gamma.\cos\phi - \sin\gamma.\sin\phi) &= \frac{\kappa}{r}\frac{\partial}{\partial r}(r.R)\sin\phi \\ &\quad + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial Z}{\partial r}\right)\left[\cos\gamma.\cos\phi - \sin\gamma.\sin\phi\right] \\ \frac{1}{r}\frac{\partial(rR)}{\partial r}.\cos\phi - \kappa Z(\sin\gamma.\cos\phi &+ \cos\gamma.\sin\phi) = 0 \end{aligned}\right\} \ldots(45{\cdot}2)$$

Now all these equations hold for all values of ϕ; but $\sin\phi$ and $\cos\phi$ vary differently. Hence the co-efficients of $\sin\phi$ and $\cos\phi$ in each must when taken together separately vanish.

Therefore

$$\left(\frac{w^2}{c_1^2} - \kappa^2\right)R + \sin\gamma.\kappa\frac{\partial Z}{\partial r} = 0 \quad \ldots \quad \ldots \quad (45{\cdot}3)$$

$$\cos\gamma.\kappa\frac{\partial Z}{\partial r} = 0 \quad \ldots \quad \ldots \quad (45{\cdot}4)$$

$$\left(\frac{w^2}{c_1^2} - \kappa^2\right)\theta + \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial (r\theta)}{\partial r}\right] = 0 \quad \ldots \quad \ldots \quad (45{\cdot}5)$$

$$\left[\frac{w^2}{c_1^2} Z + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial Z}{\partial r}\right)\right]\cos\gamma = 0 \quad \ldots \quad \ldots \quad \ldots \quad (45{\cdot}6)$$

$$-\left[\frac{w^2}{c_1^2} Z + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial Z}{\partial r}\right)\right]\sin\gamma + \frac{\kappa}{r}\frac{\partial}{\partial r}(r.\,R) = 0 \ldots \quad (45{\cdot}7)$$

$$\frac{1}{r}\frac{\partial (r\,R)}{\partial r} - \sin\gamma.\,\kappa\,Z = 0 \quad \ldots \quad \ldots \quad \ldots \quad (45{\cdot}8)$$

$$-\cos\gamma.\,\kappa\,Z = 0 \quad \ldots \quad \ldots \quad \ldots \quad (45{\cdot}9)$$

As the orbit may have any phase difference $\cos\gamma$ is not necessarily zero. Also if the orbit is inclined forces must be some function of Z, hence $\kappa \neq 0$

It follows from (45·9) that $Z = 0$ ... ... (45·10)

This might have been foreseen from the symmetry.

This satisfies (45·4) also.

Then from (45·8) we have $\frac{1}{r}\frac{\partial (r\,R)}{\partial r} = 0.$

Hence $R = \frac{A}{r}$, where A is some constant (45·11)

From (45·3) we have $\left(\frac{w^2}{c_1} - \kappa^2\right) R = 0.$

As R is not always zero, $\frac{w^2}{c_1^2} - \kappa^2 = 0$ or $\frac{w}{\kappa} = \pm c_1$ ... (45·12)

Then from (45·5) $\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial (r\theta)}{\partial r}\right] = 0$

Hence $\theta = C.r + \frac{B}{r}$, where B and C are some constants (45·13)

Hence

$$\left.\begin{aligned} E_r &= \frac{A}{r}.\cos(wt + \kappa z) \\ E_\theta &= \left(Cr + \frac{B}{r}\right).\sin(wt + \kappa z) \\ E_z &= 0 \end{aligned}\right\} \quad \ldots \quad (45{\cdot}14)$$

It is obvious that E_θ must vanish at ∞. Therefore $C = 0$,

3. We can write these in the more usual forms

$$\left.\begin{aligned} E_r &= \frac{A}{r} \cdot \cos\frac{2\pi}{\lambda}(z-ut) \\ E_\theta &= \frac{B}{r} \cdot \sin\frac{2\pi}{\lambda}(z-ut) \\ E_z &= 0 \end{aligned}\right\} \quad \ldots \quad \ldots \quad (45{\cdot}15)$$

where λ represents the wavelength associated with the phase velocity u. The system is a plane wave moving along the Z-axis, with amplitude diminishing inversely as the distance from the axis increases.

4. In order to keep a constant phase, $wt + \kappa z = \text{constant}$. Hence z must vary as $-\frac{w}{\kappa} \cdot t$, and only differ by a constant. Hence the phase velocity, say u, of the system is $-\frac{w}{\kappa}$, which from (45·12) is equal to $\pm c_1$ according as the motion is upwards or downwards.

Thus the phase velocity is equal to the characteristic velocity of the medium,

$$u = c_1 = \frac{c}{\sqrt{\mu k}}. \quad \ldots \quad \ldots \quad \ldots \quad (45{\cdot}16)$$

It is also to be noted that no matter howsoever the orbit is inclined to the helical axis of symmetry i. e. the Z-axis there is no force component along this axis and there is an automatic propagation of the system along the axis of symmetry with the constant velocity $\frac{c}{\sqrt{\mu k}}$.

Accordingly the electromagnetic radionic system is identified with "light".

5. Even if R, θ and Z were not only functions of r but also of t, it is obvious from the fourth equation of (45.2) in which differentiation with respect to time does not occur that (45·8) and (45·9) still hold good. Hence Z remains zero.

6. It can be at once seen that

$$\left.\begin{aligned} \frac{\partial^2 E_r}{\partial t^2} &= \frac{w^2}{\kappa^2}\frac{\partial^2 E}{\partial z} \\ \text{and} \qquad \frac{\partial^2 E_\theta}{\partial t^2} &= \frac{w^2}{\kappa^2}\frac{\partial^2 E}{\partial z^2} \end{aligned}\right\} \quad \ldots \quad \ldots \quad (45{\cdot}17)$$

Thus the rotatory system represents an *automatic* wave propagation with the velocity $\pm\frac{w}{\kappa}$.

7. More generally, if the Z-component of the forces remains constant and they are also symmetrical round the Z-axis, then $\partial z = 0$ and

$$\frac{\partial}{\partial\theta} = 0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (45{\cdot}18)$$

Equations (43·2) then become

$$\left.\begin{aligned}
&\frac{1}{c_1^2}\frac{\partial^2 E_r}{\partial t^2} = \frac{\partial^2 E_z}{\partial z^2} && \ldots \quad \ldots \quad (45{\cdot}19)\\
&\frac{1}{c_1^2}\frac{\partial^2 E_\theta}{\partial t^2} = \frac{\partial^2 E_\theta}{\partial z^2} + \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial (rE_\theta)}{\partial r}\right] && \ldots \quad \ldots \quad (45{\cdot}20)\\
&0 = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial E_r}{\partial z}\right) && \ldots \quad \ldots \quad (45{\cdot}21)\\
&\frac{1}{r}\frac{\partial}{\partial r}(r\,E_r) = 0 && \ldots \quad \ldots \quad (45{\cdot}22)
\end{aligned}\right\}$$

From (45·19) E_r is an automatic wave propagation with velocity c_1.

From (45·22) $E_r = \frac{A}{r} f_r\,(t, z, \theta)$, which satifies (45·21).

From (45·20) it is clear that E_θ also will be an automatic wave propagation with the same velocity c_1 if $\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial (rE\theta)}{\partial r}\right] = 0$

$$\text{which gives} \quad E_\theta = \left(\frac{B}{r} + Cr\right) f_\theta(t, z, \theta).$$

$$\text{And} \quad E_z = \text{constant}.$$

8. Equations (43·2) will hold not only for the region free from the charges, but also at the two infinitesimal points of the charges if $\frac{\partial}{\partial t}(\rho v) = 0$. In the region free from charges $\rho = 0$. At the charges v should be constant. If the motion is uniform and circular, v and its components v_r, v_θ, v_z will remain constant, and the equations will be satisfied.

SECTION V

The Rotational Theory of Matter.

1. As has been actually adopted by N. S. Japolsky, ϕ can be a complex function, for it is not absolutely necessary to take the simple case when all the coefficients are real. They can be expressed in a complex form and yet their combined result may remain real.

Consider $i\,\phi = i\,(\phi_1 + i\,\phi_2) = i\,(w_1 t + \kappa_1 z + n_1\theta) - (w_2 t + \kappa_2 z + n_2\theta)$

From (44·5) it is clear that $\sqrt{\frac{w^2}{c_1^2} - \kappa^2}$ is real.

Hence $\sqrt{\frac{w_1^2 - w_2^2}{c_1^2} - \kappa_1^2 + \kappa_2^2 + 2i\left(\frac{w_1 w_2}{c_1^2} - \kappa_1 \kappa_2\right)}$ is real

Therefore $\frac{w_1 w_2}{c_1^2} = \kappa_1 \kappa_2$ ... ... (46·1)

But $-\frac{w_1}{\kappa_1}$ represents the real phase velocity u, because in order to maintain the phase constant, z must increase with the velocity so as to keep $w_1 t + \kappa_1 z = 0$

Hence $u = -\frac{w_1}{\kappa_1}$ ... ... (46·2)

2. The solutions of Maxwell's equations hold for z referred to the origin O and the axes O_x, O_y, O_z. If axes be taken in the system moving with it relatively to O along the Z-axis, and v be the velocity of such relative motion.

then $z = \xi + vt$

Substituting we get

$$\left.\begin{aligned} \phi_1 &= w_1 t + \kappa_1 (\xi + vt) + n_1\theta \\ \phi_2 &= w_2 t + \kappa_2 (\xi + vt) + n_2\theta \end{aligned}\right\} \quad \ldots \quad \ldots \quad (46·3)$$

Now if ϕ_2 were to vary with time, the function will cease to be periodic. But the moving binary system is perfectly stable.

Hence $w_2 + \kappa_2 v = 0$

$\therefore$ $v = -\frac{w_2}{\kappa_2}$ ... ... (46·4)

From (46·1), (46·2) and (46·4) we get $u.v = c_1^2$ ... ... (46·5)

This can be identified with the well-known DE BROGLIE'S formula that the product of the velocity of the associated De Broglie wave and the velocity of the particle is equal to the square of the velocity of light in a vacuum. For a medium where $\mu \neq k \neq 1$ the product equals the square of the velocity in the medium.

Hence the phase velocity u is identified with that of the mysterious, and obscure De Broglie waves.

3. In the case of Light. as $u = c_1$ it follows that $u = v = c_1$

... ... (46·6)

Momentum × Wavelength $= p.\lambda = mc_1 . c_1 T = mc_1^2 . T$

$$= E.T = \frac{E}{\nu} = h \quad \ldots \quad \ldots \quad (46{\cdot}7)$$

SECTION VI

Relativity Postulates—A Misinterpretation.

We know from (46·5) that $u.v = c_1^2$, and from (46·2) $u = -\frac{W_1}{K_1}$. We may put $\kappa_1 = \frac{2\pi}{\lambda}$

Accordingly we can write

$$\phi_1 = \frac{2\pi}{\lambda}(z - ut) + n\theta$$

$$= \frac{2\pi}{\lambda}\left(z - \frac{c_1^2}{v}t\right) + n\theta \quad \ldots \quad \ldots \quad (47{\cdot}1)$$

Also from (46·3)

$$\phi_1 = \kappa_1\left[\xi - (u - v)t\right] + n\theta$$

$$= \frac{2\pi}{\lambda}\left[\xi - \left(\frac{c_1^2}{v} - v\right)t\right] + n\theta$$

$$= \frac{2\pi}{\lambda}\left[\xi - \frac{c_1^2}{v}\left(1 - \frac{v^2}{c_1^2}\right)t\right] + n\theta$$

$$= 2\pi\left(\frac{\sqrt{1 - \frac{v^2}{c_1^2}}}{\lambda}\right)\left[\frac{\xi}{\sqrt{1 - \frac{v^2}{c_1^2}}} - \frac{c_1^2}{v}\left(\sqrt{1 - \frac{v^2}{c_1^2}}.t\right)\right]$$

... ... (47·2)

Comparing (47·1) and (47·2) it is seen at once that the quantity ϕ_1 will retain exactly the same form and can be assumed to be an invariant whether (1) we express it with reference to fixed axes for which z stands, or (2) express it with reference to the axes moving relatively to them, for which stands ξ, *provided* in the latter case we alter the measuring units of Time and Space in such a way as to treat the new units as

$$\sqrt{1-\frac{v^2}{c_1^2}}\,.\,t' = t, \text{ and } \sqrt{1-\frac{v^2}{c_1^2}} = l' \qquad (47\cdot3)$$

These are the famous Lorentz transformations. It is obvious that with these altered units Maxwell's equations remain unchanged. This circumstance can be *wrongly* interpreted as if for a system moving with a velocity, the time expands and the length contracts. With due deference to the conclusion of Japolsky, *the real truth is that if a system is both rotating and moving forward with a uniform velocity, then vector quantities, which are subject to Maxwell's equations, do change, but can be wrongly assumed to remain an invariant if Lorentz transformations were applied. But this would not hold in the case of a simple translatory motion.*

It was shown in Chapter XIII Sec. II that the Space-Time continuum of the General Theory of Relativity is a partial misinterpretation of a spherical wave propagation. It is now clear that the postulates of Special Relativity are a partial misinterpretation of the effect of a rotatory cum translatory motion.

SECTION VII

Quantum or Wave Mechanics—A mere Rotational Mechanics.

1. We may consider any binary system, e. g. a hydrogen atom, with its two components of equal and opposite charges, rotating round each other, with their common centre of gravity moving forward with a uniform velocity v. In such a system the period of a complete revolution T can remain constant, while v can be different on ejection.

The wave length associated with the velocity of the centre of gravity given by $\lambda = vT$ is in a submicroscopic system incapable of

measurement. But the wave-length associated with the phase velocity $\lambda = u\,T$ can be measured, where T is a constant.

Now $$u.v = c_1^2 = \frac{c^2}{\mu k}$$

Let the linear momentum of the whole system along the Z axis be p and the total mass m.

Then $$p.\lambda = mv.uT = \frac{mc^2}{\mu k}.\ T = \frac{mc^2}{\mu k}.\ \frac{1}{\nu} = h \quad \ldots \quad \ldots \quad (48{\cdot}1)$$

This is the well-known relation assumed by DE BROGLIE, which is here not assumed, but actually deduced.

In the case of light, mc^2 equals the energy (see Ch. XIV, para. 5 p. 360). If. with EINSTEIN. we put the constant here also as E, the energy,

then $$E = mc^2 \quad \ldots \quad \ldots \quad \ldots \quad (48{\cdot}2)$$

PLANCK'S assumption $E = h\nu$ then follows automatically. $\ldots$ (48·3)

It is quite wrong to confuse energy with mass. In the electromagnetic free rotatory binary system, which is also moving forward, the kinetic energy is the sum total of their kinetic energies obtained from their translatory as well as rotatory motions. In Newton's Mechanics kinetic energy $= \frac{1}{2} mv^2$, which varies with v. In the rotatory system the forward velocities of the two components are so related with their circumferential velocities that the combined kinetic energy remains constant.

As r and θ are not separately measured, and the value repeats itself after each revolution we can at a long distance take the average value of

$$f(r)\, e^{i n \theta} = A \text{ as constant.}$$

Hence $$E = A\ e^{2\pi i (z - ut)} \quad \ldots \quad \ldots \quad \ldots \quad (48{\cdot}4)$$

If the axis of symmetry, along which the system is propagated, be not taken as the z-axis, but any direction r. with direction cosines (l', m', n') then the equation when referred to new fixed Cartesian coordinates becomes

$$\psi = A\, e^{2\pi i \left(\frac{mv}{h}\right)\left\{l'x + m'y + n'z - \frac{c_1^2}{v} t\right\}} \quad \ldots \quad (48{\cdot}4)$$

where ψ has been substituted for E, because the same result holds for any vector quantity of the same type, which is subject to Maxwell's equations.

3. We now get from (49·4)

$$\left.\begin{aligned} \frac{\partial\psi}{\partial x} &= \frac{2\pi i}{\lambda}\, l'\psi \quad ; & \frac{\partial^2\psi}{\partial x^2} &= \left(\frac{2\pi i}{\lambda}\right)^2 l'^2.\ \psi \\ \frac{\partial\psi}{\partial y} &= \frac{2\pi i}{\lambda}\, m'\psi \ ; & \frac{\partial^2\psi}{\partial y^2} &= \left(\frac{2\pi i}{\lambda}\right)^2 m'^2\,\psi \\ \frac{\partial\psi}{\partial z} &= \frac{2\pi i}{\lambda}\, n'\psi \ ; & \frac{\partial^2\psi}{\partial z} &= \left(\frac{2\pi i}{\lambda}\right)^2 n'^2\,\psi \\ \frac{\partial\psi}{\partial t} &= -\frac{2\pi i}{\lambda}\, u\,\psi \ ; & \frac{\partial^2\psi}{\partial t^2} &= \left(\frac{2\pi i}{\lambda}\right)^2 .\ u^2\,\psi \end{aligned}\right\} \quad \ldots\ \ldots \quad (48\cdot5)$$

$$\text{Hence } \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} - \frac{1}{u^2}\,\frac{\partial^2\psi}{\partial t^2} = 0\ , \text{ as } l'^2 + m'^2 + n'^2 = 1$$

$$\ldots \qquad \ldots \qquad \ldots \quad (48\cdot6)$$

which is a wave propagation with velocity u.

Also

$$\left.\begin{aligned} \frac{\partial\psi}{\partial x} &= \frac{2\pi i m}{h}\cdot v_x\,\psi & \frac{\partial}{2\pi i}\,\frac{\partial}{\partial x} &= m\,v_x = p_x \\ \frac{\partial\psi}{\partial y} &= \frac{2\pi i m}{h}\ v_y\,\psi & \frac{h}{2\pi i}\,\frac{\partial}{dy} &= m\ v_y = p_y \\ \frac{\partial\psi}{\partial z} &= \frac{2\pi i m}{h}\ v_z\,\psi & \frac{h}{2\pi i}\,\frac{\partial}{\partial z} &= m\,v_z = p_z \\ \frac{\partial\psi}{\partial t} &= -\frac{2\pi i m}{h}\, c_1^2\,\psi & \frac{h}{2\pi i}\,\frac{\partial}{\partial t} &= m\,c_1^2 = E \end{aligned}\right\} \quad \ldots \quad (48\cdot7)$$

It should however be remembered that these are *purely artificial devices* for representing momenta and energy, and do not in fact represent reality.

4. From (49·5) we also have

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \left(\frac{2\pi i}{\lambda}\right)^2 .\ \psi\,(l'^2 + m'^2 + n'^2) = \left(\frac{2\pi i}{\lambda}\right)^2 \psi$$

$$\text{Hence } \nabla^2\psi + \left(\frac{2\pi}{\lambda}\right)^2 \psi = 0 \qquad \ldots \qquad \ldots \quad (48\cdot8)$$

This is identified with the famous wave equation of HEISENBERG and SCHRODINGER.

SECTION VIII

The Moving Atom

1. As the period of rotation is T, the rotation of each component in the projection on the x y plane may be taken to be in the ellipse (a, b) so that

$$x = a\cos\left(\frac{2\pi}{T}t+\alpha\right) \text{ and } y = b\cos\left(\frac{2\pi}{T}t+\beta\right)$$

Hence

$$\frac{dx}{dt} = -\frac{2\pi a}{T}\sin\left(\frac{2\pi}{T}t+\alpha\right) \text{ and } \frac{dy}{dt} = -\frac{2\pi b}{T}\sin\left(\frac{2\pi}{T}t+\beta\right)$$

Therefore the mean energy of the two during the complete period is

$$W = \frac{1}{T}\int_0^T \frac{m}{2}\left[\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2\right]dt = \frac{m\pi^2}{T^2}(a^2+b^2). \quad \ldots \quad (49\cdot1)$$

2. If the components of v be v_x, v_y and v_z, then

$$v_x = l'v = l'.\frac{h}{m\lambda} \; ; \; v_y = m'.\frac{h}{m\lambda} \; ; \text{ and } v_z = n'.\frac{h}{m\lambda}.$$

Now if $\bar{E}$, V and K stand respectively for the total energy of the system, the potential energy of the electron, and the kinetic energy of the translatory motion of the whole system, and $\bar{E}' = \bar{E} - W$, then from the principle of the conservation of evergy

$$E' - V = K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m(v_x^2+v_y^2+v_z^2).$$

$$= \tfrac{1}{2}\left(\frac{h}{m\lambda}\right)^2 \text{, where m is the total mass of the atom.}$$

Hence

$$\frac{8\pi^2 m}{h}(\bar{E}'-V) = \left(\frac{2\pi}{\lambda}\right)^2 \quad \ldots \quad \ldots \quad (49\cdot2)$$

Accordingly from (48·8) we get

$$\nabla^2\psi + \frac{8\pi^2 m}{h^2}(\bar{E}'-V)\psi = 0$$

or

$$\left[\nabla^2+\frac{8\pi^2 m}{h^2}(\bar{E}'-V)\right]\psi = 0 \quad \ldots \quad \ldots \quad (49\cdot3)$$

This is similar in form to the famous HEISENBERG-SCHRODINGER equation. As for a freely moving atom $(E-V)$ is constant, this is the equation of a plane wave. Accordingly atoms will show diffraction as has already been observed by *Dempster*.

3. The falsity of the artificial devices becomes exposed when the interaction of n atoms is considered. Heisenberg-Schrodinger's equation then becomes

$$\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial y_1^2}+\frac{\partial^2}{\partial z_1^2}\right)+\left(\frac{\partial^2}{\partial x_2^2}+\frac{\partial^2}{\partial y_2^2}+\frac{\partial^2}{\partial z_2^2}\right)+\ldots$$

$$+\left(\frac{\partial^2}{\partial x_n^2}+\frac{\partial^2}{\partial y_n^2}+\frac{\partial^2}{\partial z_n^2}\right)+\frac{8\pi^2 m}{h^2}(E-V)\Big]\psi=0$$

which represents an imaginary wave propagation in a space of $3n$ dimensions.

4. The Rotational Theory steers clear of such an impossible mathematical fiction. If any number of rotating systems have the same wave length and the same phase velocity, then they can be easily compounded.

$$E_r=\sum_1^n E_{nr}=\sum_1^n\left\{R_n\,.\,e^{\frac{2\pi i}{\lambda}(z-ut+\alpha_n)}\right\}$$

$$=R\,.\,e^{\frac{2\pi i}{\lambda}(z-ut+\alpha)} \qquad \ldots \quad \ldots \quad \ldots \quad (49{\cdot}4)$$

where $$R=\sqrt{(\Sigma R_n\cos\alpha_n)^2+(\Sigma R_n\sin\alpha_n)^2}$$

and $$\tan\alpha=\frac{\sum_1^n R_n\sin\alpha_n}{\sum_1^n R_n\cos\alpha_n}.$$

We get similar values for E_y and E_z.

SECTION IX

Rotating Magnet.

1. A glance at the equations (43·2) and (43·3) would show that they are exactly identical. Hence if we take

$$M = f(r)\, e^{i(wt + kz + n\theta)}$$

we obtain for the magnetic intensity exactly the same form as in the preceding Sections. Hence a rotating magnetic diapole will produce a similar field as a rotating electric diapole.

2. Thus if we have a rotating magnet, with its positive and negative poles we get the same result. If the distance between the poles is variable they must be connected by some sort of an elastic band. But if the distance between them is not variable, they would each describe circles round the common centre of gravity, even if the latter is not equidistant from the two poles. In such a case there would be no question of any rotation of the orbits, and the results would be identical with those in Sec. IV with $n = 0$.

Thus a light radion (though this is not probable) can just as well be a rotating magnet, instead of a binary system with its two component charges separated.

3. It will be seen that not only the whole system, but each component is subject to the equations (43.2). Hence all the results of Sec. IV apply to a single electron moving round a proton, because the force acting on it is always radial. A glance at the figure in Ch. XIV, Sec. VI, P. 358 will show that even a single rotating electron can emit an electromagnetic wave.

The effect of a spinning magnet, behaving in a similar way, will be considered later in connection with the diffraction of free electrons for which the potential energy = 0.

APPENDICES

A. Appendix to Ch. II. Sec VI para 3.

MICHELSON AND MORLEY'S EXPERIMENT

It was pointed out there that for this experiment monochromatic light produced on the Earth has been used so far, and that like ordinary material particles it may already possess the velocity of the earth in addition to its own.

Hence in the experiment $\frac{1}{(c-v)+v}+\frac{1}{(c+v)-v}\equiv\frac{1}{c}+\frac{1}{c}$

It follow that any experiment in which *terrestrial* light is used is inconclusive and does not therefore prove the postulate of Relativity. To have a crucial test, it is necessary to use light coming from outside the earth, so that its velocity would be independent of that of the Earth.

Although in view of (47·3) and the physical explanation suggested in Ch. IV. Sec. II para. 2 p. 230, it is just possible that the velocity of light may be wholly independent of that of its source yet that is not likely.

AN EXPERIMENT SUGGESTED

Although for proving interference, it is not absolutely necessary to have monochromatic light, such light can conveniently be obtained, which is not produced on the Earth.

Light from the Sun should first be passed through a series of prisms (as in a monochromator), then all other light, except that with the chosen wavelength, intercepted by obstacles, and only nearly monochromatic light allowed to fall on a plane reflector and then directed into the Michelson-Morley Apparatus. Although the intensity will be diminished, there will be a greater facility to observe the interference fringes.

AN APPEAL

I earnestly appeal to experimenters to repeat the experiment with Solar light which will test whether light is corpuscular in character and also whether the postulate of Relativity is not sound. I venture to make the PREDICTION that *there will no longer be the null effect.*

B. Appendix to Ch. XIII Sec. II p. 278

ADDENDUM

In May last an Addendum to Ch. XIII Sec. II p. 278 was printed and circulated, pointing out that although P, Q and R must be symmetrical functions of x,y and z, it had been unnecessarily thought that i Ddt also must come in similarly. On reconsideration it seemed that this may not be so. The reason is that although the two equations of the gravitational wave are symmetrial with regard to x, y, z, and w, the orbital equation of the particle, unaffected by gravitation is not so because of v.

Without any such restriction as to symmetry, if the gravitational influence is superimposed, the orbital equation would be

$$(1+f_1)\,dr^2 + (1+f_2)\,r^2\,d\theta^2 + (f_3 - v^2)\,dt^2 = 0 \quad \ldots \quad \ldots \quad (50{\cdot}1)$$

where the f's are some unknown functions. From symmetry round the origin they must be independent of θ; and as the influence would be proportional to the gravitating mass M at the origin, they must contain M as a factor; remembering that they must vanish at ∞ and expanding them by Laurent's theorem we get

$$\left(1 + M\sum_1^\infty \frac{A_n}{r^n}\right) dr^2 + \left(1 + M\sum_1^\infty \frac{B_n}{r^n}\right) r^2\,d\theta^2 + \left(M\sum_1^\infty \frac{C_n}{r^n} - v^2\right) dt^2 = 0.$$

where, the A's, B's and C's are constants and are known to be small. Snbstituting $r^2\frac{d\theta}{dt} = h$, and retaining terms of the order $\frac{1}{r^3}$ only we get

$$\left(\frac{dr}{r^2 d\theta}\right)^2 + \left(1 + \frac{MB_1}{r}\right)\frac{1}{r^2} = \frac{v^2}{h^2} - \frac{MC_1}{h^2}\cdot\frac{1}{r}$$

or
$$\left(\frac{du}{d\theta}\right)^2 + (1 + MB_1 . u)\,u^2 = \frac{v^2}{h^2} - \frac{MC_1}{h^2}\cdot u$$

Hence
$$\frac{d^2u}{d\theta^2} + u = -\frac{MC_1}{2h^2} - \frac{3MB_1}{2}\cdot u^2$$

$$= \frac{GM}{h^2} + \frac{3GM}{D^2}\cdot u^2 \quad \ldots \quad (50{\cdot}2)$$

where $G = -\frac{C_1}{2}$ is identified with the gravitational constant, and $D = +\frac{C_1}{B_1}$ is another universal constant found to be equal to the velocity of light.

ADDENDUM

to Sec. IV, para. 2, page 74.

Even without the particular case that $C = 0$, the solution

$$E_\theta = (Cr + \frac{B}{r}) \sin(\omega t + kz)$$

can admit of a physical explanation.

As r increases, E_θ does not approach infinity, but a limit is reached when the attractive force is so much diminished that the components part company and proceed on parabolic paths. The binary system therefore merely ceases to exist beyond this limit.

Again, as r diminishes, a limit is reached when the force of attraction is so great that the two components rush towards each other and coalesce, and the binary system ceases to exist. Inside this limit the binary rotating system cannot be preserved without an extremely large transverse force ; and as such large force does not exist the binary system itself breaks down.

Thus we have physical explanations of Sir J. J. Thomson's otherwise inexplicable "two cores". (*Nature* Vol. 137 pp. 232-3)

References

1. Eddington : *Nature of the Physical World*, p. 179.
2. Einstein : *The World As I See It*, p. 138.
3. Padolsky : *Phil. Mag. Vol.* XXII, p. 998.
4. Sulaiman : *Ind. Phy-Math. Journ.* VIII p. 12.
5. Japolsky : *Phil. Mag. Vol.* XIX pp. 93-4.
6. Do. : *Ibid. Vol* XX pp. 417-468.

ON A SIMPLE DERIVATION OF STRESSES IN A MOVING FLUID

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Received November 26, 1936

SUMMARY

This note describes a simple method of deriving equations for internal forces in a fluid which is in relative motion, paying special attention to the physical principles involved.

Stokes[1] derived the equations for stresses in a fluid which was in motion by considerations of internal friction and showed that the pressures which remain the same in all directions in the case of motionless fluid, vary in different directions when internal friction is present. His method of derivation though lengthy is very instructive. While Lamb[2] and others have obtained the same formulæ by a process which does not throw any light into the internal mechanism of the shifting motion and the friction that results from such a type of fluid motion. Take for instance a plane sound wave travelling along x axis, the pressure p_1 in the same direction is given by

$$p_1 = -p + \tfrac{2}{3}\mu\triangle + 2\mu\dot{e}_1 \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

where p represents the hydrodynamical pressure, $\dot{\triangle}$ the rate of volume dilatation and $\dot{e}_1$ the time rate of extension of fluid elements along x axis. In the present case

$$\dot{\triangle} = \dot{e}_1$$

and $\dot{e}_1 = \dfrac{du}{dx}$, where u represents the fluid velocity along x axis.

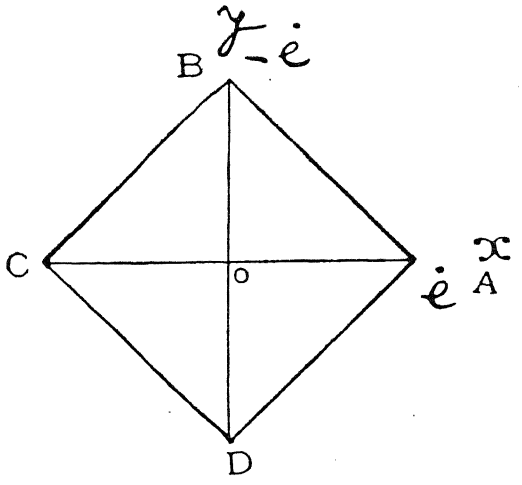
According to the definition of viscosity the tangential friction between two fluid layers in motion is given by

$$T = \mu\dot{\theta}$$

and $$\dot{\theta} = \frac{du}{dy} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

where $\dot{\theta}$ is the rate of shearing strain.

Thus from (2) we find that tangential friction arises when there is a gradient of u along y axis ; while (1) shows that the internal friction has entered into the formula when there is no variation of u along y axis, the wave being plane $\frac{du}{dy}=0$. Lamb's general formulæ could not throw any light on this point. And it was this difficulty that led the writer to investigate and derive the general formulæ from the fundamental definition i. e., equations (2) with the help of the principle of superposition of shifting motions laid down by Stokes.



Let us suppose that a cube of fluid is isolated from the rest so that A B C D represents its section when it is at rest. Further let the motion of the fluid element take place in such a manner that extensions of elements paralle to O A and OB are parallel to x,y axes respectively, while there is no motion of fluid elements parallel to OZ, ie Z axis. Hence we can consider the deformation of the two dimensional square A B C D and find out the stresses that arise during motion. If the time rate of extensions of elements along OA (x axis) and O B (y axis) be $\dot{e}$ and $-\dot{e}$ then we know that these give rise to shifting motion along AB and we have

$$\theta' = 2\dot{e} \quad T = 2\mu e \qquad \ldots \qquad \ldots \quad (3)$$

and while the normal stress across planes perpendicular to OA and OB along x and y axes are P and $-P$, and

$$P = T \qquad \ldots \qquad \ldots \qquad \ldots \quad (4)$$

If we put $P/e = a$ where a is at present an unknown constant, then from (3) and (4) we find $a = 2\mu$

thus
$$\left.\begin{aligned} P_1 &= 2\mu\, e \text{ along x axis} \\ P_2 &= -2\mu\, e \text{ along y axis} \end{aligned}\right\} \quad \ldots \quad (5)$$

In the case when there are shifting motions along other directions also, the relation between the time rate of extensions and stresses can be obtained by the principle of superposition. Let us consider three cases represented in Figures (2) (3) and (4) which represent sections of the elemental cube of fluid in xy, xz, and zy planes respectively.

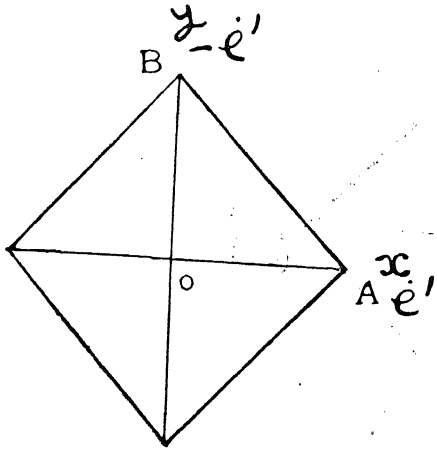


Fig. (2)

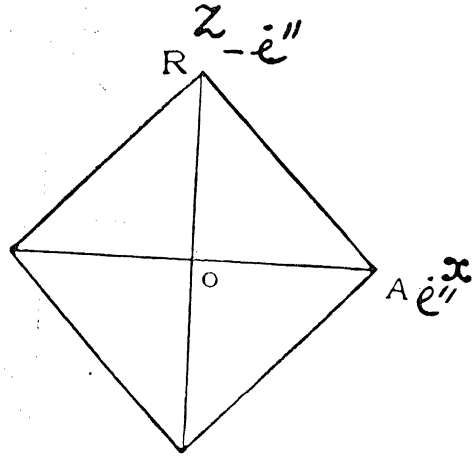


Fig. (3)

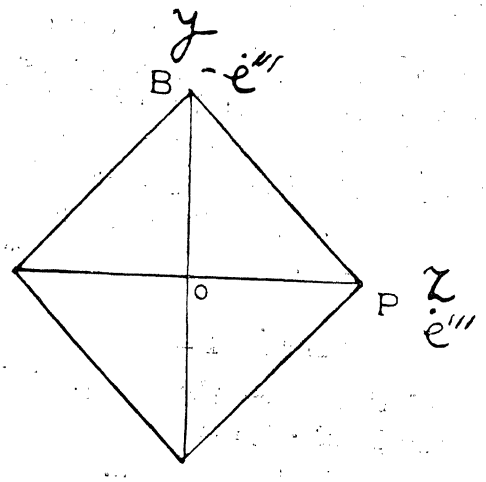


Fig. (4)

Let $\dot{e}'$ and $-\dot{e}'$ represent the rate of extensions of fluid elements along x and y axes for shifting motion parallel to A B Fig (2) so that all planes parallel to A B and perpendicular to the plane of the paper shift relatively to A B and give rise to a rate of shear of amount

$$\dot{\theta}_1 = 2\dot{e}'$$

Similarly let extensions alongOX and OZ Fig. (3) be $\dot{e}''$ and $-\dot{e}''$ which give rise to shifting motion parallel to AR; while Fig (4) represents extensions along OZ, and OY of magnitudes $\dot{e}'''$ and $-\dot{e}'''$ produce shifting motion along P B. These together represent the most general type of shifting motion that might occur in a moving fluid.

If X_x, Y_y and Z_z represent normal stresses along x, y, and z axes respectively across planes for which x, y and z have constant values,

then $X_x = 2\mu(\dot{e}' + \dot{e}'')$, $Y_y = 2\mu(-\dot{e}'-\dot{e}''')$,

and $Z_z = 2\mu(\dot{e}'''-\dot{e}'')$ by (5) ... ... (6)

And $\dot{e}_x = (\dot{e}'+\dot{e}'')$, $\dot{e}_y = -(\dot{e}'+\dot{e}''')$, and $\dot{e}_z = (\dot{e}''' - \dot{e}'')$ ... ... (7)

where $\dot{e}_x$, $\dot{e}_y$ and $\dot{e}_z$ represent the total extensions along x, y and z axes. Further we observe that

$$\dot{e}_x + \dot{e}_y + \dot{e}_z = 0$$

as should be the case in the case of pure shifting motions, hence for pure shifting motions the rate of dilatation $\dot{\delta} = 0$

Equations (6) can be written down as

$$Xx = 2\mu\dot{e}_x \; Y_y = 2\mu\dot{e}_y \; Z_z = 2\mu\dot{e}_z \quad \ldots \quad \ldots \quad (9)$$

If $\dot{e}x = \dot{e}y = \dot{e}z$ we have a rate of uniform dilatation. It will be shown presently that such a rate of dilatation does not produce any stress which depends upon the internal friction of fluid, that is, μ. From (7) we find that when $\dot{e}_x = \dot{e}_y = \dot{e}_z$

$$\dot{e}' = -\dot{e}'', \text{ and } \dot{e}' = -\dot{e}'''$$

and $X_x = Y_y = Z_z = 0$ ... ... (10)

It is therefore evident that from the general type of rates of extensions

$\dot{e}_x$, $\dot{e}_y$ and $\dot{e}_z$, the portion which is due to rate of uniform dilatation should be subtracted. Let us assume that $\dot{e}_x = \dot{e}'_x + \dot{e}$, $\dot{e}_y = \dot{e}_y' + \dot{e}$, $\dot{e}_z = \dot{e}_z' + \dot{e}$ where $\dot{e}'x$, $\dot{e}y'$ and $\dot{e}z'$ represent the rate of extensions for pure shifting motion and $\dot{e}$ the extension for uniform dilatation.

Putting $\dot{e}_x + \dot{e}_y + \dot{e}_z = \dot{\triangle}$ the dilatation, we observe that

$$\dot{\triangle} = \dot{e}_x' + \dot{e}_y' + \dot{e}_z' + 3\dot{e} = \dot{\delta} + 3\dot{e}$$

But by (8) $\dot{\delta} = 0$, hence $\dot{e} = \dot{\triangle}/3$

and
$$\left.\begin{aligned} X_x &= 2\mu(\dot{e}_x - \dot{\triangle}/3) \\ Y_y &= 2\mu(\dot{e}_y - \dot{\triangle}/3) \\ Z_z &= 2\mu(\dot{e}_z - \dot{\triangle}/3) \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

Now we are in a position to write down the equations for stresses when the hydrodynamical pressure p is also present. This pressure is the same in all direction, hence considering the forces on the face of a cube of fluid,

we find
$$P_1 = -p + X_x$$
$$P_2 = -p + Y_y$$
$$P_3 = -p + Z_z$$

where P_1, P_2 and P_3 represent the total stress along x, y and z axes respectively. Substituting the values of X_x, Y_y and Z_z from (11)we get

$$\left.\begin{aligned} P_1 &= -p - \tfrac{2}{3}\mu\dot{\triangle} + 2\mu\dot{e}_x \\ P_2 &= -p - \tfrac{2}{3}\mu\dot{\triangle} + 2\mu\dot{e}_y \\ P_3 &= -p - \tfrac{2}{3}\mu\dot{\triangle} + 2\mu\dot{e}_z \end{aligned}\right\} \quad \ldots \quad \ldots \quad (1)$$

These are the fundamental equations for stresses in a fluid which is in and when viscosity is taken into consideration. The mean stress, motion $(P_1 + P_2 + P_3)/3 = -p$, the same as the hydrodynamical pressure.

We shall now illustrate our formulæ by a discussion of the case referred to in the introductory paragraphs.

Since there is no motion along y and z axes, $\dot{e}_y = \dot{e}_z = 0$ and $\dot{e}_x = \dot{e}_1$; $\dot{\triangle} = \dot{e}_1$, while $\dot{e} = \dot{e}_1/3$ by (11). Thus

$$P_1 = -p - \tfrac{2}{3}\mu\dot{e}_1 + 2\mu\dot{e}_1$$

$$P_2 = -p - \tfrac{2}{3}\mu\dot{e}_1$$

$$P_3 = -p - \tfrac{2}{3}\mu\dot{e}_1$$

And referring to equations (7) and (11) we find in the present case

$$\dot{e}_x' = \dot{e}_x - \frac{\dot{\triangle}}{3} = \dot{e}_1 - \frac{\dot{e}_1}{3} = \tfrac{2}{3}\dot{e}_1 = (\dot{e}' + \dot{e}'')$$

Similarly $\quad -\dot{e}' - \dot{e}''' = -\dot{e}_1/3$

and $\quad \dot{e}''' - \dot{e}'' = \dot{e}_1/3.$

From these relations we evaluate $\dot{e}'$, $\dot{e}''$ and $\dot{e}'''$. It is found that

$$\dot{e}''' = 0\ ,\ \dot{e}' = \dot{e}_1/3, \text{ and } \dot{e}'' = \dot{e}_1/3, \text{ and } \dot{e} = \dot{e}_1/3$$

Thus we find that this kind of motion is equivalent to (1) a rate of uniform dilatation of magnitude $\dot{\triangle} = \dot{e}_1$, and the shifting motions produced by extensions $\dot{e}' = \dot{e}_1/3$ along x axis, with an equivalent contraction along y axis; and another pair of extensions of magnitude $\dot{e}'' = \dot{e}_1/3$ along x axis, and $-\dot{e}_1/3$ along z axis. (Refer to Figs. 2 and 3). These two shifting motions with a rate of uniform dilatation produce the requisite motion $\dot{e}_1$ along x axis only, for

$$\dot{e}_x = \dot{e}'_x + \dot{\triangle}/3 = \dot{e}' + \dot{e}'' + \dot{e}_1/3$$

$$= \frac{\dot{e}_1}{3} + \frac{\dot{e}_1}{3} + \frac{\dot{e}_1}{3} = \dot{e}_1$$

$$\dot{e}_y = -\frac{\dot{e}_1}{3} + \frac{\dot{e}_1}{3} = 0$$

$$\dot{e}_z = -\dot{e}'' + \dot{\triangle}/3$$

$$= -\frac{\dot{e}_1}{3} + \frac{\dot{e}_1}{3}$$

$$= 0$$

Thus all the conditions of problem are satisfied.

References

1, Lamb, H. (1914) *Hydrodynamics*, 571-577.

2. Stoke, G. G. (1897) *Collected Works*, **1**, 1-20.

REFLECTION OF RADIO WAVES FROM THE IONOSPHERE

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(Communicated by Prof. M. N. Saha)

Received January 30, 1937

SUMMARY

Assuming that considerable reflection of radio waves takes place in the ionosphere at a place where electronic density is such that $\frac{d\mu}{dN}$ is very large, it has been shown that the ordinary wave can be reflected from only one concentration of electrons while the extraordinary wave has in general three conditions of reflection for values of $p > p_H$.

Our experimental result that the penetration frequency difference between the extraordinary and the ordinary rays has on several occasions been observed to be only ·14mc/sec. when the frequency of the incident waves was about 4 mc/sec has been explained in the light of the theoretical results obtained.

It has been shown that reflection of radio waves from the ionosphere can take place not only from the places where group velocity is zero and phase velocity infinite but also from the strata where both the velocities are together zero.

Introduction

Since the year 1901, the problem of the propagation of radio waves has been attracting much attention and within the last few years a large amount of work has been done on the propagation of these waves through the upper ionized regions under the influence of the earth's magnetic field. The magneto-ionic theory developed by Breit, Appleton, Goldstein and Hartree has many notable successes to its credit. Appleton [1] generalized the treatment given by Lorentz and obtained the general expressions for dispersion and polarization of the waves. The same result was obtained by Hartree [4] in an altogether different manner. Martyn [5] has drawn dispersion and polarization curves for a number of wave lengths taking different values of collisional frequency and three values of θ (0°, 45° and 90°) the angle between the direction of the earth's magnetic field and

the direction of propagation of the waves. Mary Taylor [8,9] has also drawn the same curves with and without friction though only for Slough (England). All these investigators have assumed that the reflection of waves takes place from the place where refractive index becomes zero. Booker[2], however, in his theoretical exposition of the various important aspects of the problem has mentioned that appreciable reflection of waves will take place when either the complex refractive index q becomes approximately equal to zero, or the change of q per vacuum wave length is considerable. He has neglected the latter condition remarking that it would be applicable to long waves and the whole of his treatment is based on the former condition only. The following analysis will show that the second condition applies not only to long waves but to short waves as well and that it is in fact the general condition which contains the former condition as a special case.

Theoretical

We know that the refractive index changes its value with the variation in the electron density N. It is also well known that in the ionosphere there exists an ionization gradient, i. e. the ionization density increases with height. Thus the condition that 'when change in the refractive index per vacuum wave length is considerable', may very well be taken to be the same as 'when change in refractive index for small changes in the value of N, the ionization density, is large', or to put it mathematically when $\frac{d\mu}{dN}$ is large. As $p_0^2 = \frac{4\pi Ne^2}{m}$, we can also take the condition of reflection as $\frac{d\mu}{d(p_0^2)} = \infty$. We shall here consider only the simple case when friction is neglected. The refractive index μ in the absence of friction and polarisation term is given by

$$\mu^2 = 1 - \frac{2p_0^2(p^2 - p_0^2)}{2p^2(p^2 - p_0^2) - p^2 p_T^2 \pm \sqrt{p^4 p_T^4 + 4p^2 p_T^2 (p^2 - p_0^2)^2}} \quad \ldots (1)$$

where, μ = Refractive Index.

p = pulsatance $= 2\pi \times$ frequency.

$p_{L,T} = \frac{eH_{L,T}}{mc}$.

e = charge on an electron.

m = mass of an electron.

$H_{L,T}$ = vertical and horizontal component of the earth's magnetic field.

c = Velocity of light in vacuum.

$$p_0^2 = \frac{4\pi Ne^2}{m}.$$

N = Number of electrons per c. c.

Differentiating with respect to p_0^2 and simplifying a little we get

$$\frac{d\mu}{d(p_0^2)} = -\frac{[2p^2(p^2-p_0^2)-p^2p_T^2 \pm \sqrt{p^4p_T^4+4p^2p_L^2(p^2-p_0^2)^2}]}{[2p^2(p^2-p_0^2)-p^2p_T^2 \pm \sqrt{p^4p_T^4+4p^2p_L^2(p^2-p_0^2)^2}]^{\frac{3}{2}}}$$

$$\frac{(2p^2-4p_0^2)+2p^2p_0^2(p^2-p_0^2)[1 \pm \frac{2p_L^2(p^2-p_0^2)}{\sqrt{p^4p_T^4+4p^2p_L^2(p^2-p_0^2)^2}}]}{[2(p^2-p_0^2)^2-p^2p_T^2 \pm \sqrt{p^4p_T^4+4p^2p_L^2(p^2-p_0^2)}]^{\frac{1}{2}}} \quad \ldots \quad \ldots \quad (2)$$

Now in order that $\frac{d\mu}{d(p_0^2)}$ may be infinitely large either,

$$2p^2(p^2-p_0^2)-p^2p_T^2 \pm \sqrt{p^4p_T^4+4p^2p_L^2(p^2-p_0^2)^2} = 0 \quad \ldots \quad (3)$$

or, $$2(p^2-p_0^2)^2-p^2p_T^2 \pm \sqrt{p^4p_T^4+4p^2p_L^2(p^2-p_0^2)^2} = 0 \quad \ldots \quad (4)$$

Taking the positive sign in equation (4) we get

$$p_0^2 = p^2 \quad \ldots \quad \ldots \quad (5)$$

It is obvious that condition (5) appears to make (2) of the form $\frac{0}{0}$ but when we find the limit of the function as $p \to p_0$ we notice that it comes out to be infinity and hence (5) is a condition of reflection. The negative sign in equation (4) gives,

$$p_0^2 = p^2 + pp_H \quad \ldots \quad \ldots \quad (6a)$$

$$p_0^2 = p^2 - pp_H \quad \ldots \quad \ldots \quad (6b)$$

Again, when we consider equation (3) and take the positive sign, we get the same condition as given by (5) while when we take the negative sign, we get,

$$p_0^2 = \frac{p^2-p_H^2}{p^2-p_L^2}\,p^2 \quad \ldots \quad \ldots \quad (7)$$

Thus we see that this method of looking at the problem gives us not only the usually accepted conditions of reflection as given by equations (5) and (6) but also a new condition represented by equation (7). The equation (5) gives the condition of reflection of what is known as the ordinary ray

and equations (6) and (7) give the conditions of reflection for the extraordinary ray. There is an interesting result from equation (7) that from the same concentration of ionization two waves of different frequencies can be returned and obviously the condition is applicable only to waves whose pulsatance p is either greater than p_H or less than p_L, while condition (6b) is applicable only to waves of pulsatance greater than p_H. We thus come to the conclusion that for waves of pulsatance greater than p_H there is only one concentration of ionization from where the ordinary ray can be reflected while there are, in general, three concentrations from which the extra-ordinary ray can be sent back.

It is easy to see that if we impose a further condition $\mu=0$ upon the general condition of reflection $\frac{d\mu}{d(p_0^2)}=\infty$, we get only one concentration of electrons for the reflection of the ordinary ray and two for that of the extraordinary ray. Thus the general condition of reflection is that at the place of reflection $\frac{d\mu}{dN}$ is very large of which the condition $\mu=0$ is only a special case.

Same condition has been obtained both by Goubau[3] and by Rai[7] proceeding on the assumption that at the place from where radio waves are reflected, group velocity of the incident waves becomes zero.

Quasitransverse propagation

This case shall be obtained from the general case by putting p_L equal to zero and will represent the conditions at the places situated along the magnetic equator. Putting p_L equal to zero in equations (3) and (4) we obtain,

$$2(p^2-p_0^2)^2-p^2p_T^2 \pm p^2p_T^2 = 0 \quad \ldots \quad \ldots \quad (8)$$

$$2p^2(p^2-p_0^2)-p^2p_T^2 \pm p^2p_T^2 = 0 \quad \ldots \quad \ldots \quad (9)$$

The positive sign in both the equations gives,

$$p_0^2 = p^2$$

while the negative sign in (8) gives,

$$p_0^2 = p^2 \pm pp_T$$

and that in (9) gives,

$$p_0^2 = p^2 - p_T^2$$

Thus we see that in this case also we obtain one condition of reflection for the ordinary wave while in general there are three conditions for the exraordinary ray. But if the pulsatance p of the waves is less than p_T, there is only one condition of reflection for extraordinary ray.

Quasilongitudinal propagation

This case shall be obtained by putting $p_T = 0$ in the general case and shall apply to places situated near the magnetic poles. By making the above substitution in equations (3) and (4), we see that (3) does not give any useful result while (4) gives only two conditions of reflection,

$$p_0^2 = p^2 \pm pp_L$$

Thus we see that in this case the results are exactly the same as the ones obtained by the usual method of obtaining the conditions of reflection by putting refractive index equal to zero.

Experimental Results

For the last several months we have been studying the F_2 region. It is generally seen that during the day, except in the early morning when ionization is building up, and in the early part of the night only one of the two magneto-ionic components is visible, (the other being absorbed). Late at night, however, both components are visible and disappear at different frequencies. As already reported[6] on many occasions it was found that if the extraordinary ray disappeared at 4 mc/sec the ordinary one penetrated at about 3·86 mc/sec. Thus the frequency difference was about ·14 mc/sec This result was very perplexing for it could not be explained on the existing theory. But it can be very easily explained on the results obtained in the previous section. It appears that when such low values of critical penetration frequency differences are obtained the extraordinary ray gets reflected from the electron concentration level given by equation (7) while ordinary ray is reflected from the usual level given by equation (5). If f_1 is the critical frequency at any time for the extraordinary ray while f_2 is that for the ordinary ray, using equations (7) and (4) we get,

$$\frac{p_1^2 - p_H^2}{p_1^2 - p_L^2} p_1^2 = p_2^2 \quad \ldots \quad \ldots \quad (10)$$

where $p_{1,2} = 2\pi f_{1,2}$

from which $f_1 - f_2$ for Allahabad comes out to be ·142 mc/sec if f_1 is 4 mc/sec.

Putting condition (10) in the form

$$p_1 - p_2 = \frac{p_T^2}{(1 - p_L^2/p_1^2)\,(p_1 + p_2)}$$, it is clear that as critical frequency increases the difference will decrease and it will show a diurnal variation, being least when the ionization reaches its maximum and greatest when it is minimum while the frequency difference $p_1 - p_2$ obtained by equation (5) and (6) comes out to be a constant quantity approximately $p_{H/2}$ independent of the value of the critical penetration frequency. On some occasions we have also observed a frequency difference of ·64 mc/sec, when the frequency of the waves sent up was about 4mc/sec, This is easily explained from the usually accepted condition of reflection for the extraordinary ray given by equation (6). This value has been reported by investigators all over the world but no mention seems to have been made by any worker of the previous value.

Phase and Group Velocities

Now we will show that the new condition obtained shows that reflection under these circumstances takes place from a place where phase velocity becomes zero. It will also be seen that at this place both phase and group velocities become zero.

The standard formula for group velocity is,

$$\frac{1}{U} = \frac{1}{c}\left(p\frac{d\mu}{dp} + \mu\right)$$

substituting the value of μ and $\frac{d\mu}{dp}$ from equation (1), we have

$$\frac{U}{c} = \frac{N}{D}$$

where, $N = \left[2p^2\,(p^2 - p_0^2) - p^2\,p_T^2 \pm \sqrt{p^4 p_T^4 + 4\,p^2\,p_L^2\,(p^2 - p_0^2)^2}\,\right]^{\frac{3}{2}}$

$$\left[2(p^2 - p_0^2)^2 - p^2\,p_T^2 \pm \sqrt{p^4\,p_T^4 + 4\,p^2\,p_L^2\,(p^2 - p_0^2)^2}\,\right]^{\frac{1}{2}}$$

$$\text{and } D = \Big[-\Big\{ 2p^2 (p^2 - p_0^2) - p^2 p_T^2 \pm \sqrt{p^4 p_T^4 + 4p^2 p_L^2 (p^2 - p_0^2)^2} \Big\} 4p^2 p_0^2$$
$$+ 4 p^2 p_0^2 (p^2 - p_0^2) \Big\{ 4p^2 - 2p_0^2 - p_T^2$$
$$\pm \frac{p^2 p_T^4 + 4p^2 p_L^2 (p^2 - p_0^2) + 2p_L^2 (p^2 - p_0^2)^2}{\sqrt{p^4 p_T^4 + 4 p^2 p_L^2 (p^2 - p_0^2)^2}} \Big\}$$
$$+ \Big\{ 2p^2 (p^2 - p_0^2) - p^2 p_T^2 \pm \sqrt{p^4 p_T^4 + 4 p^2 p_L^2 (p^2 - p_0^2)^2} \Big\}$$
$$\times \Big\{ 2(p^2 - p_0^2)^2 - p^2 p_T^2 \pm \sqrt{p^4 p_T^4 + 4 p^2 p_L^2 (p^2 - p_0^2)^2} \Big\} \Big]$$

Again the phase velocity V is given by

$$\frac{V}{c} = \frac{1}{\mu} = \left\{ \frac{2p^2 (p^2 - p_0^2) - p^2 p_T^2 \pm \sqrt{p^4 p_T^4 + 4p^2 p_L^2 (p^2 - p_0^2)^2}}{2(p^2 - p_0^2)^2 - p^2 p_T^2 \pm \sqrt{p^4 p_T^4 + 4 p^2 p_L^2 (p^2 - p_0^2)^2}} \right\}^{\frac{1}{2}}$$

Now it is easy to see that when the reflection conditions given by equations (5) and (6) are fulfilled group velocity is zero and phase velocity is infinite, while when the condition (7) is satisfied both phase and group velocities are zero.

My heartiest thanks are due to Prof. M. N. Saha, D. Sc., F. R. S., and Dr. G. R. Toshnival, D. Sc., for their keen interest and useful discussions. My best thanks are also due to Mr. R. N. Rai for going through the calculations.

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LATTICE ENERGIES OF SOME ALKALI IODIDES AND THE ELECTRON AFFINITY OF IODINE

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Received April 17, 1937

SUMMARY

By studying the dissociation equilibrium of the vapours at high temperatures, the lattice energies of KI and NaI have been determined to be 150·6 and 166·4 K. Cals, respectively. The same experimental arrangement, as described in the work on the electron affinity of chlorine by M. N. Saha and the author has been utilised. By the application of the Born's Cycle to the lattice energy determinations the electron affinity of iodine has been found to be 72. 4 K. Cals. The results are in good agreement with the calculations made from Born's theory and other experimental data.

Introduction

Unlike the other halogens, the electron affinity of iodine has been the subject of investigation of various workers who used different methods. In the present paper an attempt has been made to confirm the older results on the electron affinity of iodine from the measurements of the lattice energies of sodium and potassium iodides. The first investigation on the measurement of electron affinity of the halogens was made by Angerer and Muller [1] who, following an idea of Franck, observed that the halogen gases begin to absorb continuously with a long wave length limit. They attributed the sharp long wave length limit to absorption by X and calculated the electron affinity. Although the results which they obtained are in good agreement with the results obtained from theoretical considerations, yet the interpretation is not free from doubt. A more direct method was used later by J. E. Mayer [9] who determined the lattice energies of the iodides of caesium and potassium and calculated the electron affinity of iodine. In the present investigation the same method has been applied with modified and improved experimental technique. In 1935 P. P. Sutton and J. E. Mayer[13] published another work in which they described a different method for the determination of electron affinity and applied it to the case of iodine. In this method iodine vapour was introduced into an evacuated glass globe in which a tungsten filament surrounded by a concentric anode cylinder was heated. The iodine vapour molecules on coming into contact with the strongly heated filament suffered dissociation into atoms which captured electrons liberated from the filament. The currents due to the negatively charged iodine atoms and electrons could be measured with the help

of the surrounding cylinder. An electromagnetic trap was used to distinguish between the electron and ion currents. Knowing the magnitudes of the ion and electron currents the dissociation constant of the reaction ($Ig_{as} + e = I^- g_{as}$) was determined from which the electron affinity of iodine was calculated. In another method used by G. Glockler and M. Calvin [4] the concentration of iodine ions and electrons was measured from their effect on space charge due to the difference in the masses of the ions and electrons.

On the theoretical side the most important work has been done by Prof. Max Born and J. E. Mayer[1] who have obtained a formula for the lattice energies of ionic crystals by a modification of the older theory. Their formula was used by L. Helmholtz and J. E. Mayer [5] to calculate the lattice energies of the alkali halides, and recently M. L. Huggins[7] has recalculated the lattice energies by taking into account more recent data. In the following table a summary of the results on the lattice energies of alkali-iodides and of the electron affinity of iodine is given.

Table 1

Salt	Lattice Energy (K. Cals.)			Electron affinity of iodine from lattice energy measurements (K. Cals.)			Electron affinity of iodine by other methods (K. Cals.)	
	Theoretical	By direct experiment	By spectroscopic method	Theoretical calculation	By direct experiment	Spectroscopic method	By G. Glockler and M. Calvin	By P. P. Sutton and J. E. Mayer.
LiI	174·1[6] 176·1[7]			75·8[6] 73·1[7]				
NaI	163·9[6] 164·3[7]	166·4*	166	73·9[6] 73·5[7]	72·0*	71·3	74·6 ± 1·5	72·4 ± 1·5
KI	150·8[6] 152·4[7]	153·8[9] 150·6*	151	73·2[6] 71·6[7]	72·62[9] 73·4*			
RbI	145·3[6] 148·0[7]		148	73·8[6] 71·1[7]				
CsI	139·1[6] 142·5[7]	141·5		74·2[6] 70·8[7]				

* PRESENT PAPER

THEORY

At high temperatures the process of dissociation of any alkali iodide MI can be represented by the following set of equations :—

$$MI = M + I - D \quad \ldots\ldots (1)$$

$$MI = M^+ + I^- - Q \quad \ldots\ldots (2)$$

$$M = M^+ + e - I_c \quad \ldots\ldots (3)$$

$$I = I + e - E_x \quad \ldots\ldots (4)$$

$$I_2 = 2I - D' \quad \ldots\ldots (5)$$

where D is the heat of dissociation of the alkali halide, Q the heat of dissociation into ions, I_c the ionisation potential of the alkali, E_x the electron affinity of iodine and D′ the heat of dissociation. If K_2 is the dissociation Constant of the reaction (2).
then we have the formnla.

$$\log K_2 = \log \frac{p_{M^+} p_{I^-}}{p_{MI}} = -\frac{Q}{2 \cdot 3RT} + \tfrac{3}{2}\log T + \log (1 - e^{-h\nu/KT}) + \log \left[\frac{\kappa^{\frac{3}{2}}}{2^{\frac{3}{2}} \pi^{\frac{1}{2}} Ih} \left(\frac{m_M m_I^{\frac{2}{2}}}{m_{MI}} \right) \right]$$

Also if L_{MI} be the latent heat of evaporation then the lattice energy $\phi(r_o)$ is given by $\phi(r_o) = Q + L_{MX}$ and $E = D + I_o - Q$. If therefore K_2 is known experimentally at any temperature Q can be determined from which Ex can be calculated.

EXPERIMENT

The demountable vacuum furnace used in this experiment has already been described in a previous paper[10], and the internal arrangements have also been described in another communication[11]. The salt was heated in an electric furnace and the vapour was made to enter the high temperature region of the graphite tube where dissociation according to equations (1)—(5) occurred. The products of dissociation passed through a limiting diaphragm and were finally collected by a Faraday cylinder

connected to a cell and galvanometer. The Faraday cylinder was given a small positive or negative potential of 2 volts with respect to the graphite tube in order to measure the negative or positive current. If T is the temperature of the graphite tube, p_{MI} the pressure of the salt, i_{I^-} the current due to the negative ions, the dissociation constant K_2 is given by

$$K_2 = \frac{p_{M^+} \cdot p_{I^-}}{p_{MI}} = \frac{i_{M^+} \cdot i_{I^-}}{e^2 S^2} (2\pi kT) \frac{\sqrt{m_M m_I}}{p_M}$$

where S is the area of the effusion hole, K the Boltz-mann constant, m_M the mass of alkali atom in absolute units and m_I the mass of the iodine atom. The total current I_{M^+} is related to the measured current I_g by the relation

$$i_{M^+} = \frac{2d^2}{r^2} \quad i_G$$

where d is the distance between the effusion hole and the limiting diaphragm.

The temperature of the outer surface of the graphite tube was measured by a disappearing filament type of pyrometer. It was suspected* that this temperature did not give the true equilibrium temperature which is somewhat greater than this and in order to verify this another experiment was performed. A Pt-Rh thermo-couple which was re-calibrated in the laboratory at the melting points of K Cl, Na Cl, and copper was inserted inside the graphite tube, and could be utilised for measuring the temperature of the inner side of the tube. The outer temperature was also measured by the pyrometer. These observations were repeated at several temperatures and a curve was plotted with the outer temperature of the graphite tube against the inner temperature.

*This factor has been neglected in the author's previous works on chlorine[11] and bromine.[13] Those results have been re-calculated now by the author and B. N. Srivastava after making this temperature correction.

Results

In tables 2 and 3 the results obtained for KI and NaI are given. The vapour pressure of KI has been calculated from the empirical formula

$$\log p_{mm} = -\frac{8229}{T} + 8{\cdot}0957$$

due to Fioch and Rodebush[3], and for Na I, von Wartenburg and S-hulz's[14] formula.

$$\log p_{dynes} = -\frac{37000}{4{\cdot}57T} + 11{\cdot}1355$$

has been used.

The magnitudes of the moments of inertia[6] and the characteristic frequency[8] of vibrations* for the two molecules KI and NaI are

I	355×10^{-40} ,	169×10^{-40}
$\frac{h\nu}{k}$	303	408

The lattice energy for KI has been found to be 150·6 K-Cals; and for NaI 166 ± K-Cals. The heats of dissociation of KI and NaI are 76·3 and 72·3 K-Cals respectively and the heats of ionisation of potasium and sodium are 99·5 and 117·3 K-Cals. These data therefore give an average value of 72·4 K-Cals for the electron affinity of iodine.

The contribution of this in the term $\log (1 - e^{-h\nu/kT})$ was not taken into account in the author's previous works on chlorine and bromine, although it contributes about 7 to 8% in the result. This error was unfortunately balanced in neglecting the temperature correction mentioned before and hence the results were not much affected. The previous results have been recalculated now by the author and B. N. Srivastava.

Table 2

Diameter of the effusion hole (mms)	Vapour pressure (dynes/cm²)	Temperature of the outer surface measured by the pyrometer (°C).	Equilibrium temperature obtained from calibration curve	$i_{K^+} \cdot 10^5$ (amps)	$i_{I^-} \cdot 10^5$ amps)	$e^{-\frac{h\nu}{KT}}$	$K_2 \times 10^4$ (dynes)	Q. K ( )	$\phi_{(ro)}$ K-Cals.
1·86	36·19	1600	1680	35·16	26·69	·8563	269·6	101·2	149·4
,,	43·41	1550	1625	21·05	15·00	·8525	72·51	103·4	151·6
,,	52·91	1545	1615	25·07	17·58	·8517	83·7	102·2	150·4
,,	43·41	1445	1510	10·39	7·498	·8437	16·99	102·0	150·2
,,	62·9	1425	1485	9·228	5·768	·8417	16·97	100·6	148·8
1·106	116·3	1550	1625	25·93	9·262	·8525	88·49	102·5	150·7
,,	27·52	1515	1585	·02	3·167	·8494	45·73	101·9	150·1
,,	43·41	1500	1570	6·395	4·541	·8484	52·53	101·4	149·6
,,	90·69	1475	1540	6·275	4·243	·8461	22·68	102·2	150·4
,,	62·9	1450	1515	3·287	2·331	·8441	92·76	104·5	152·7
,,	27·52	1450	1515	2·331	1·673	·8441	10·79	104·0	152·2
,,	62·9	1410	1470	2·749	2·151	·8405	58·09	103·0	151·2

Average of lattice energy = 150·6 K-cal

$E_l = (76·3 + 99·5 - 102·4)$

= 73·4 K-Cals.

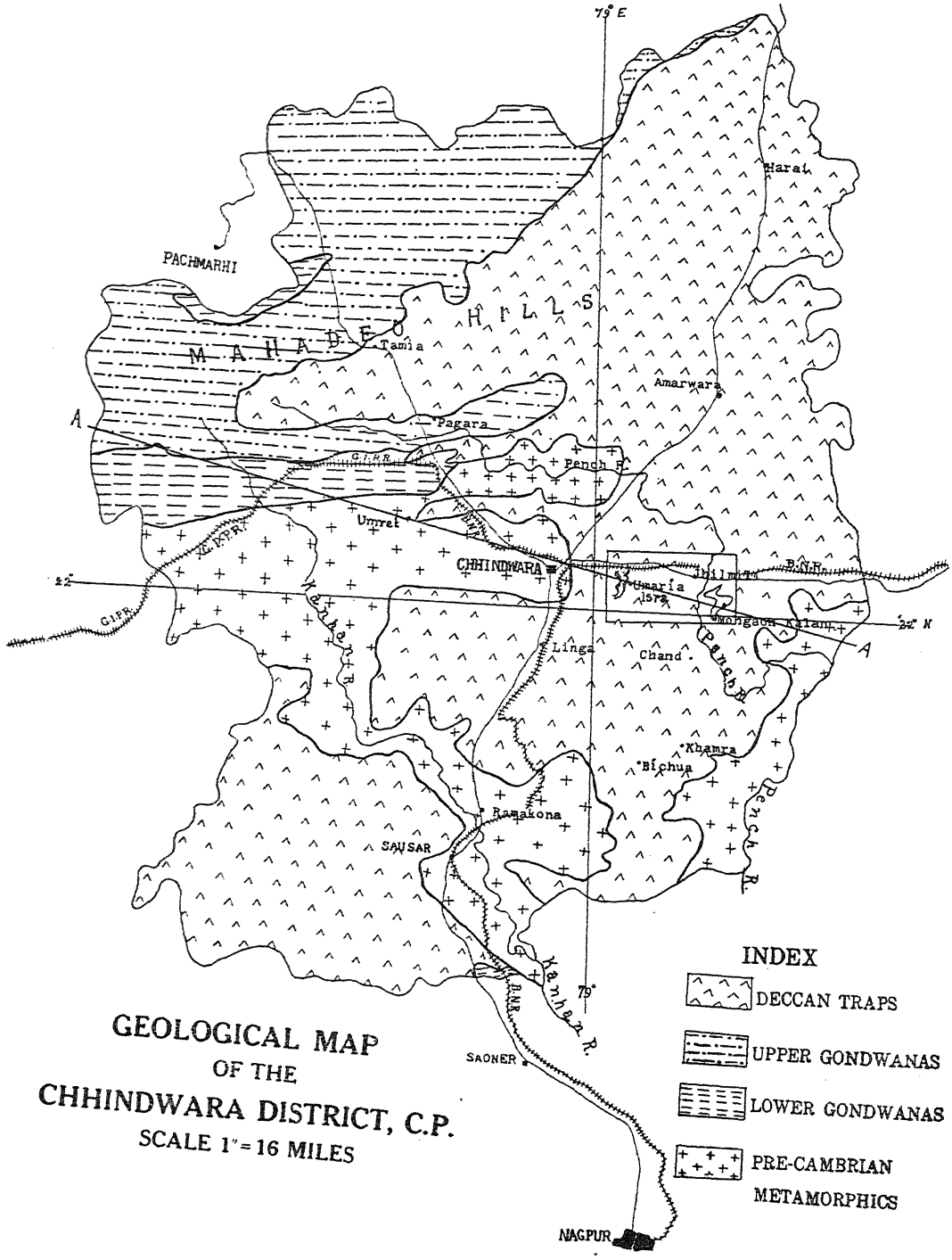
Table 3

Diameter of the effusion hole (mms)	Vapour pressure-dynes/cm.^{2}	Temperature of the outer side of the graphite-tube (°C)	Equilibrum temperature from the calibrated curve (°C)	$i_{Na^+} \times 10^6$ (amps)	$i_{I^-} \times 10^6$ (amps)	$e^{-\frac{h\nu}{KT}}$	$K_2 \times 10^6$	Q. K-Cals	$\phi_{(ro)}$ K-Cals
1·29	32·25	1575	1655	23·43	10·45	·811[illegible]	838·3	120·7	168·3
,,	56·56	1550	1625	24·39	12·44	·8066	188·1	118·5	166·1
,,	18·73	1500	1570	11·71	5·387	·8013	111·9	116·8	164·4
,,	28·21	1480	1550	12·19	5·834	·7995	81·56	116·6	164·2
,,	45·34	1460	1525	10·29	5·404	·7971	38·84	117·8	165·4
,,	114·7	1410	1470	7·508	4·542	·7918	9·156	119·1	166·7
,,	82·13	1410	1470	6·216	3·825	·7918	8·915	119·1	166·7
1·102	67·38	1600	1680	26·3	15·78	·8114	398·9	119·1	166·7
,,	38·68	1570	1650	15·3	10·04	·8050	255·6	119	166·7
,,	23·42	1505	1575	6·23	3·347	·8019	·54·46	119·9	167·5
,,	67·38	1500	1570	9·563	5·26	·8013	46·69	120	167·6
,,	38·68	1475	1540	5·626	3·211	·7986	26·19	119·8	167·4

Average of lattice energy = 166·4 K-Cals.

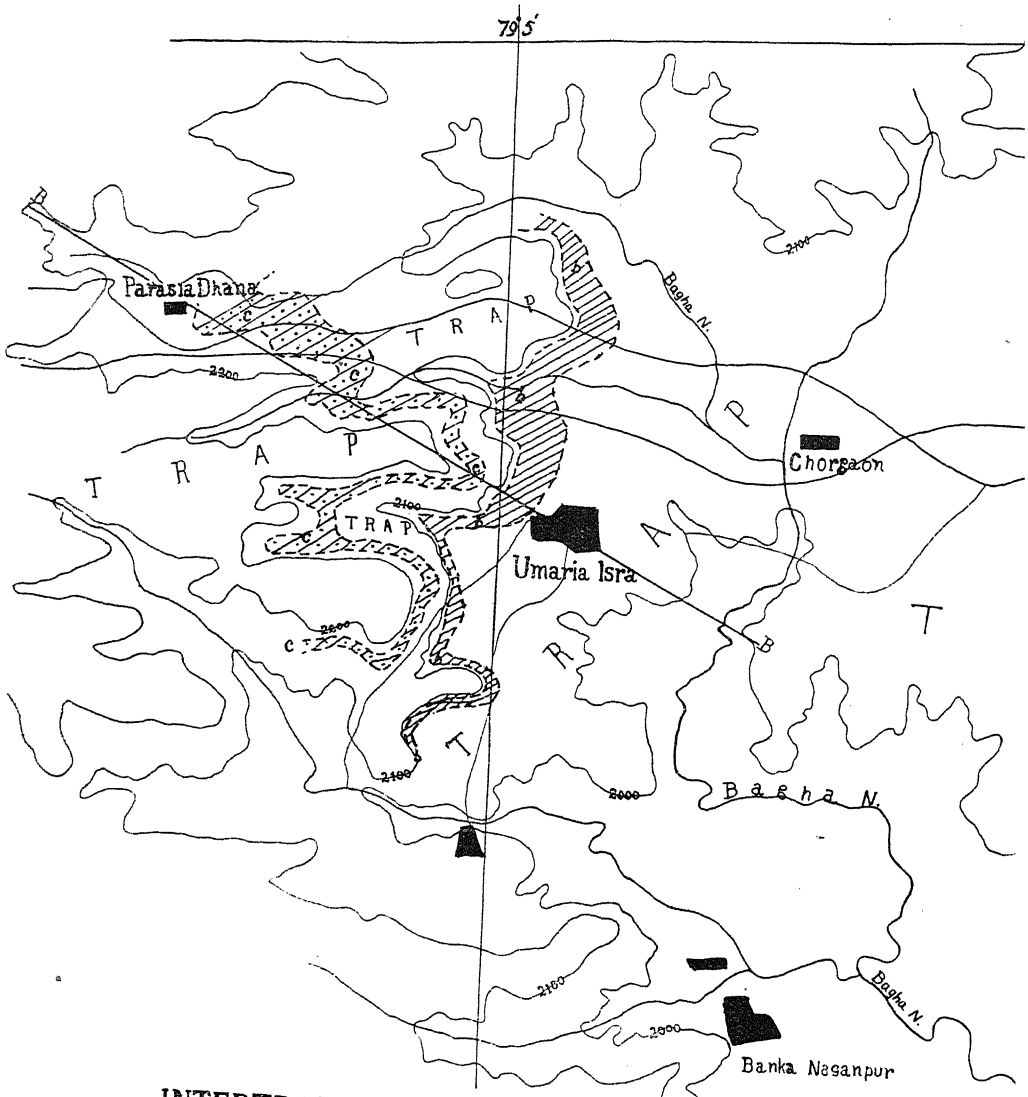
$E_1 = 72·3 + 117·9 - 118·8$

$= 72·4$ K-Cals.



GEOLOGICAL MAP
OF THE
CHHINDWARA DISTRICT, C.P.
SCALE 1" = 16 MILES

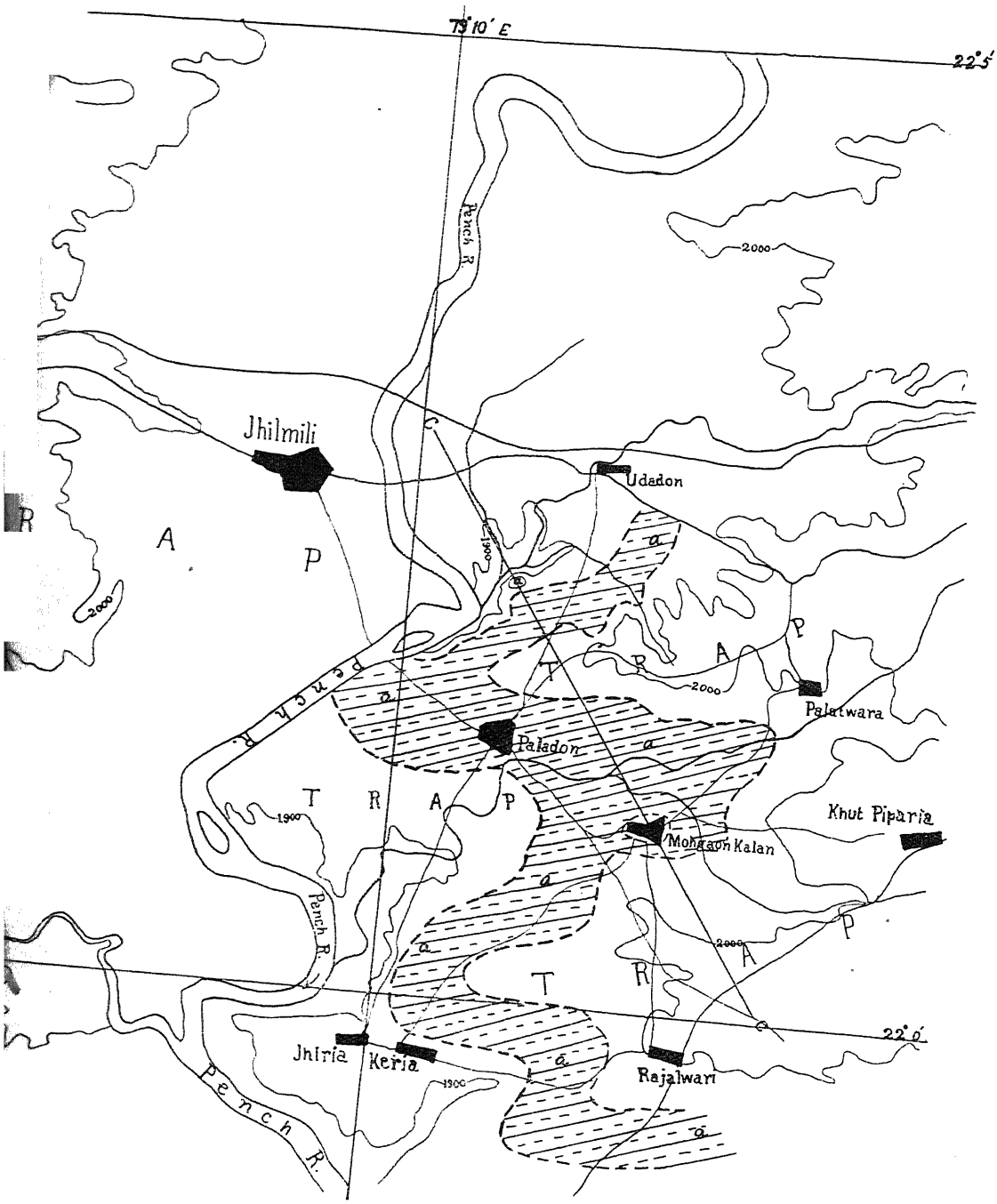
Fig. 1

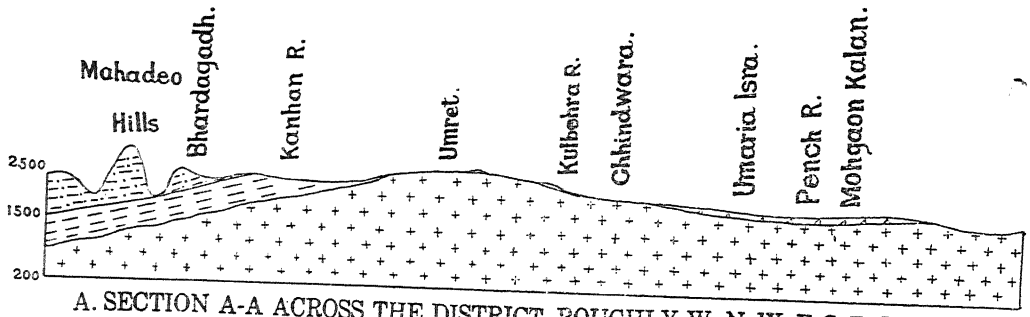


INTERTRAPPEAN BEDS OF UMARIA ISRA & MOHGAON KALAN

SCALE 1"=4 MILES

Shaded portion represents outcrops of intertrappean horizons. a, b, c,


75°10' E
22°5'
Jhilmili
Udadon
Palatwara
Paladon
Mohgaun Kalan
Khut Piparia
Jhiria
Keria
Rajalwari
22°0'
Panch R.
2000
1900
T R A P

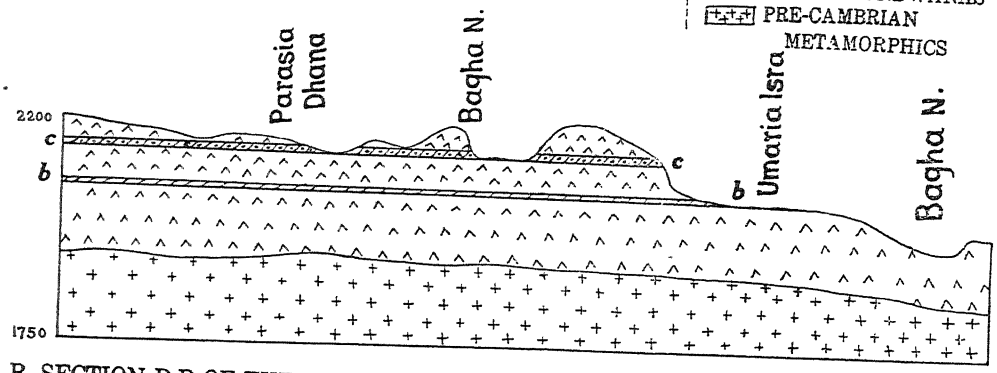


A. SECTION A-A ACROSS THE DISTRICT ROUGHLY W. N. W.-E. S. E. PASSING THROUGH CHHINDWARA TOWN UMARIA ISRA AND MOHGAON KALAN.

HORIZONTAL SCALE 1"=16 MILES

VERTICAL SCALE 1"=4,000 FT.

DECCAN TRAPS
UPPER GONDWANAS
LOWER GONDWANAS
PRE-CAMBRIAN METAMORPHICS

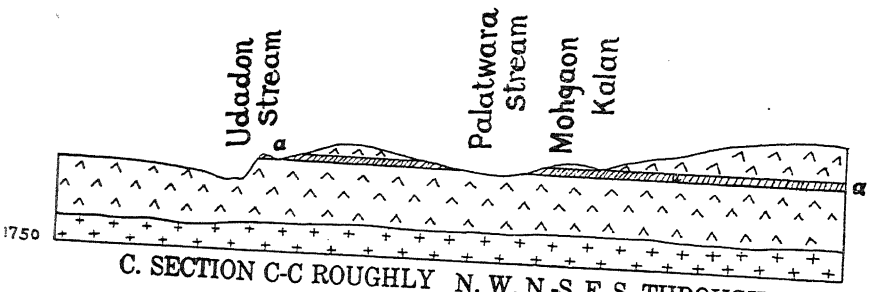


B SECTION B-B OF THE HILLS NEAR UMARIA ISRA

HORIZONTAL SCALE 1"=1 MILES

VERTICAL SCALE 1"=4 00 FT.

C
B
} INTERTRAPPEAN BEDS



C. SECTION C-C ROUGHLY N. W. N.-S. E. S. THROUGH MOHGAON KALAN

HORIZONTAL SCALE 1"=1 MILE

VERTICAL SCALE 1"=4 00 FT.

A INTERTRAPPEAN BEDS

Fig. 3

Discussion of results

It can be seen from table 1 that the results obtained are in good agreement with theoretical claculations as well as with the experimental value obtained by other workers. Theoretically the currents due to the positive nad negative ions as given in tables 2 and 3 should be inversely proportional to the square root of their atomic weights, but it will be seen that the magnitude of the current due to I⁻ is always slightly greater than its theoritical value. The same effect was observed in the author's previous work on the lattice energy of potassium and sodium bromides. It was pointed out there that this is due to the presence of free electrons which are thermally liberated from the graphite tube at high temperature In their work on the lattice energies of RbBr and NaCI, L. Helmoltz and J. E. Mayer[6] observed that the experimental ratio of the two currents was different from the theoretical and found an excess of the positive current. According to them the abnormal increase of one particular type of ion current is due to the formation of an ion sheath of the opposite sign round the effusion hole. The experimental conditions under which they have worked are very much different from those in the present work. In the present investigation the order of temperatures used is very high and therefore the former explanation is more probable. It can be shown that probable error in the experiment does not exceed 3 K. cals. The value of the electron affinity of iodine is therefore $72{\cdot}4 \pm 3$. K. cals.

In the end I wish to express my sincere thanks to Prof. M. N. Saha, D.Sc., F.R.S. for his guidance and encouragement throughout the work, and to Mr. B. N. Srivastava for helping in the calibration of thermo-couples and obtaining the curve. Our thanks are also due to the Royal Society of London for giving a grant which enabled us to construct the furnace and buy its accessories.

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TWO NEW FISH TREMATODES FROM ALLAHABAD

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Communicated by Dr. H. R. Mehra

Received May 6, 1937

SUMMARY

The present paper deals with two apparently new species of fish trematodes belonging to the genera *Fleurogenes* Looss, 1896 and *Opegaster* Ozaki, 1928 which were collected from two different host species, namely, *Callichrous pabda* and *Gobius giuris*, examined at Allahabad.

Pleurogenes pabdai n. sp.

Descriptions of species belonging to this genus from India appear in the works of Lühe (1901), Klein (1905), Mehra and Negi (1928) and Srivastava (1934). These trematodes have been found to parasitize three species of *Rana*. This genus has for the first time been encountered in a fish in this country.

Habitat—Small intestine of *Callichrous pabda*.

Description :—Body elliptical, only slightly flattened dorsoventrally, with a brown patch in the middle due to ripe eggs, and posterior bifid extremity, measures * 1·3—1·4 in length and 0·86 in maximum breadth which lies in the testicular zone. Large scale like cuticular spines are present over the entire body. Subcuticular cells are conspicuous laterally in the preacetabular part and ventrally in the anterior half of the body. The subterminal oral sucker measures 0·2 × 0·25-0·27 in diameter. Prepharynx is present. The pharynx is nearly globular and measures 0·05-0·059 × 0·048-0·059 in size. The narrow oesophagus of about 0·12 length divides into the caeca immediately behind the anterior fourth of the body length.

* All measurements in millimetres.

The intestinal caeca, simple, wide and elongated, extend transversely to the body length terminating just in front of the testes. The acetabulum, pre-equatorial in position, is smaller than the oral sucker and measures 0·18-0·2 in diameter. The genital pore is ventral and lies to the left side near the body wall close behind the pharyngeal level. The excretory pore is situated at the base of the caudal indentation and the bladder is Y-shaped. The main stem of the bladder divides into the cornua at about the middle of the post-acetabular part of the body. The cornua extend up to the middle of the testes lying ventrally to them.

The two large elongated and smooth testes lie symmetrically near the lateral body walls close behind the caeca with their anterior half in level with the acetabulum. The right testis measures 0·3-0·32 × 0·18-0·22 and the left 0·29-0·35 × 0·18-0·2 in size. The cirrus sac is obliquely placed and lies close in front of the left testis with its posterior end immediately in front of the anterior margin of the acetabulum. It is recurved distally before it opens at the genital pore. It contains mostly the large, looped vesicula seminalis which continues terminally into a spherical pars prostatica which in its turn leads into a recurved ductus ejaculatorius. Prostate gland cells are well-developed.

The somewhat oval ovary measures 0·2-0·23 × 0·13-0·18 in size. It is situated to the right side in the oesophageal zone just in front of the testis of that side. The receptaculum seminis, 0·07-0·18 × 0·04-0·09 in size, and Mehlis' gland lie slightly to the right side of the anterior half of the acetabulum. The Laurer's canal opens on the middorsal surface close behind the acetabulum. The long, voluminous, and somewhat regularly coiled uterus occupies all the available space behind the testes and the acetabulum. The descending and the ascending parts lie to the right and the left sides respectively with the connecting loops situated transversely behind the acetabulum. Terminally the uterus lies ventral to the cirrus sac, the metraterm being absent. The ripe eggs measure 0·026-0·029 × 0·011-0·014 in size. The vitellaria, composed of 18—22 large follicles, lie dorsally occupying nearly the entire body region in front of the caeca and extending anteriorly to the middle of the oral sucker.

Remarks :—This species in the extracaecal position of the ovary resembles *P. medians* (Olsson) of Looss, 1894, *P. minus* Pigulewsky, 1931 and *P. japonicus* Yamaguti, 1936 but is separated from all of them as well as other species included in the genus in the bifid posterior end of its body. From *P. medians* it also differs in the position of its transversely placed intestinal caeca, pre-equatorial acetabulum, more anterior position of its ventral genital pore, the shape of its cirrus sac and its continuous vitellaria. It can be clearly separated from *P. minus*, the only other species recorded so far from a fish host, on account of larger body size; different sucker-ratio (acetabulum is larger than oral sucker in *P. minus*); position of intestinal caeca, genital pore, ovary ; and shape of the testes. *P. japonicus*

differs from the new species in the position of the acetabulum, marginal genital pore, ovary and testes; in the shape of its cirrus sac; and in the presence of a metraterm.

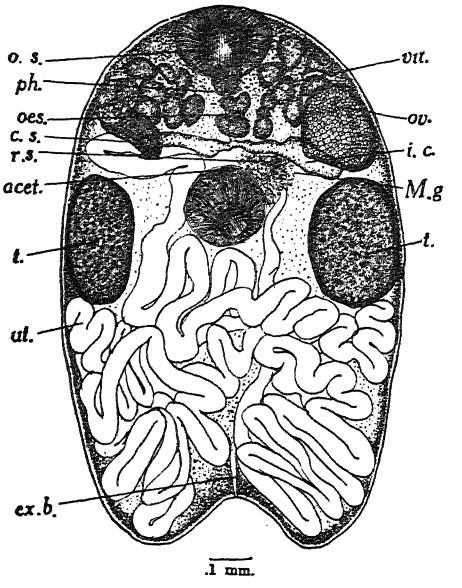

Fig. 1.—Dorsal view of *Pleurogenes pabdai* n. sp.

acet., acetabulum; *a. c.*, anal canal; *c. s.*, cirrus sac; *eg.*, egg; *ex. b.*, excretory bladder; *g. p.*, genital pore; *i. c.*, intestinal caecum; *M. g.*, Mehlis' gland; *oes.*, oesophagus; *o. s.*, oral sucker; *ov.*, ovary; *ph.*, pharynx; *r. s.*, receptaculum seminis; *t.*, testis; *ut.*, uterus; *v. s.*, vesicula seminalis; *vit.*, vitellaria; *vit. r.*, vitelline reservoir.

Opegaster beliyai n. sp.

Harshey (1933, 1937) has described three species of this genus from this country—*O. anguilli* from Allahabad and *O. mastacemballi* and *O. mehrii* from Jubbulpore. He has also given a key of species belonging to this genus in his recent paper. The present species is the fourth representative of this genus from India.

Habitat—Small intestine of *Gobius giuris.*

Description :—The body is flattened, somewhat elongated oval, with pointed anterior and rounded posterior ends and measures 1·28-2·7 in length and 0·45-0·97 in maximum breadth in the acetabular zone. The subterminal oral sucker is 0·11-0·18 in diameter. The prepharynx measures 0·03 in length. The pharynx is 0·07-0·08 long and 0·06-0·09 broad. The oesophagus, 0·11-0·23 in length, bifurcates into the caeca immediately in front of the one-fourth body length from the anterior extremity. The intestinal caeca unite to form an anal canal which opens ventrally a little in front of the posterior end of the body. The prominent acetabulum located

on a short, thick pedicel and with six marginal papillae, three on the anterior border and three on the posterior, measures 0·2-0·32 in diameter. Its centre lies just behind the first third of the body. The genital pore is situated on the ventral surface to the left side behind the middle of the oesophagus nearer the body wall. The excretory pore lies on the dorsal side a little in front of the hinder end of the body in level with the anus.

The transversely elongated testes lie directly one behind the other near the middle of the posterior body half. The anterior testis measures 0·05-0·09 × 0·29-0·34 and the posterior one 0·1-0·13 × 0·25-0·27 in size. The vesicula seminalis extends backwards to the posterior end of the acetabulum. Its terminal part before opening into the small cirrus pouch is surrounded with a few prostate gland cells.

The kidney-shaped ovary lies immediately in front of the anterior testis, measuring 0·07-0·09 × 0·16-0·22 in size. The yolk reservoir is well-developed and situated anterolateral to the ovary. The uterine coils, with 15 eggs, are transversely situated between the ovary and the acetabulum. The ripe eggs are yellow in colour and measure 0·07 × 0·037-0·04 in size. The vitelline follicles begin about the middle of the oesophagus occupying the lateral as well as the post-testicular median fields.

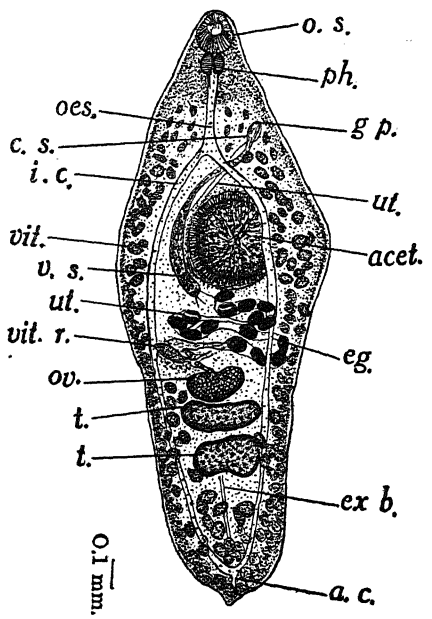

Fig. 2.—Ventral view of *Opegaster beliyai* n. sp. Lettering as in Fig. 1.

Remarks:—In having six marginal papillae on the acetabulum, the present species resembles *O. brevifistula* Ozaki 1928, *O. mehrii* Harshey 1937, *O. mastacemballi* Harshey 1937, and *O. syngnathi* Yamaguti 1934. In *O. brevifistula*

the oesophagus is absent and the acetabular papillae are much more prominent than in my species. *O. mastacemballi* is distinguished from the new species by the presence of a transverse connecting band of vitelline follicles situated close behind the acetabulum. Moreover, the former species is characterised by a shorter oesophagus and the more forward position of the vitellaria which extend up to the middle of the pharynx. *O. syngnathi* has the genital pore slightly to the right side of the intestinal bifurcations, *i.e.*, slightly posterior to the condition in the new species. Besides, in the Japanese species the anterior-most vitelline follicles are rudimentary with the result that the vitellaria appear to begin behind the acetabulum. *O. mehrii* differs from the present form in the position of the genital pore and the absence of anal canal. The description of *O. ozaki* Layman, 1930, is inadequate for assigning it under this genus. According to Yamaguti (1934) the acetabular papillae in *O. ovatus* Ozaki, 1925, reported to be absent, are six in number of which the three anterior ones are conspicuous while the three posterior ones are indistinct. *O. ovatus* is further distinguished from *O. beliyai* by sucker ratio, anterior extent of vitellaria and size of eggs.

The writer is deeply indebted to Dr. H. R. Mehra for his valuable guidance and to Dr. D. R. Bhattacharya for the facilities provided in the Department.

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ON THE CONVERGENCE OF THE CONJUGATE SERIES OF THE DERIVED FOURIER SERIES

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Communicated by Dr. B. N. Prasad

Received January 27, 1937

1. Let the series

$$\tfrac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx) \quad \ldots \quad \ldots \quad (1{\cdot}1)$$

be the Fourier series corresponding to the function $f(x)$ which is periodic with period 2π and integrable in the sense of Lebesgue. The object of this paper is to obtain a very general theorem for the convergence of the series

$$\sum_{n=1}^{\infty} n(a_n \cos nx + b_n \sin nx) \quad \ldots \quad \ldots \quad (1{\cdot}2)$$

which is the conjugate series of the Derived Fourier series corresponding to $f(x)$. It has been shown by Gupta[1] that (1·2) under certain conditions will converge to

$$-\frac{1}{\pi}\int_0^{\pi}\frac{f(x+t)+f(x-t)-2f(x)}{4\sin^2 t/2}\,dt \quad \ldots \quad \ldots \quad (1{\cdot}3)$$

at every point x at which this integral exists. The author proves in this paper that even if (1·3) ceases to exist, the series (1·2) may still converge to

$$-\frac{1}{4\pi}\int_0^{\pi}\psi(t)\,dt-\frac{1}{4\pi}\int_0^{\pi}\Psi(t).\ \cot\frac{t}{2}\ \operatorname{cosec}^2\frac{t}{2}\,dt \quad \ldots \quad (1{\cdot}4)$$

where

$$\psi(t) = f(x+t)+f(x-t)-2f(x)$$

and

$$\Psi(t) = \int_0^t \psi(t)\,dt.$$

It has been shown later on that (1·4) can exist even if (1·3) diverges but it will always exist whenever (1·3) is convergent.

2. *Theorem : The series* (1·2) *converges to the sum*

$$-\frac{1}{4\pi}\int_0^\pi \psi(t)\,dt-\frac{1}{4\pi}\int_0^\pi \Psi(t).\ \cot\frac{t}{2}\ \operatorname{cosec}^2\frac{t}{2}\,dt$$

at every point x *at which this integral exists, provided that*

(1) $\psi(t)$ *is absolutely continuous in* $(0, \pi)$,*

(2) $\frac{\psi(t)}{t}$ *is continuous and of bounded variation in* $(0, \pi)$,

(3) $\frac{\psi(t)}{t} = 0(1)$, *for small* t.

Let S_n denote the partial sum of the first n terms of the series (1·2), then we have

$$S_n = \sum_{m=1}^{m=n} m\,(a_m \cos mx + b_m \sin mx)$$

$$= -\frac{1}{4\pi}\int_0^\pi \psi(t)\ \operatorname{cosec}^2\frac{t}{2}\,dt + \frac{1}{4\pi}\int_0^\pi \psi(t)\ \operatorname{cosec}^2\frac{t}{2}\cos nt\,dt$$

$$+\frac{1}{2\pi}\int_0^\pi \psi(t)\,n\cos nt\,dt + \frac{1}{2\pi}\int_0^\pi \psi(t)\cot\frac{t}{2}\cdot n.\sin nt\,dt.$$

$$\equiv -I_1 + I_2 + I_3 + I_4 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2{\cdot}1)$$

We have

$$-I_1 + I_2 = -\frac{1}{4\pi}\int_0^\pi \psi(t)\,dt + \frac{1}{4\pi}\int_0^\pi \psi(t)\cos nt\,dt - \frac{1}{4\pi}\int_0^\rho \psi(t)\cot^2\frac{t}{2}(1-\cos nt)\,dt$$

$$-\frac{1}{4\pi}\int_\rho^\pi \psi(t)\cot^2\frac{t}{2}\,dt + \frac{1}{4\pi}\int_\rho^\pi \psi(t)\cot^2\frac{t}{2}\cos nt\,dt.$$

$$\equiv -\frac{1}{4\pi}\int_0^\pi \psi(t)\,dt + \frac{1}{4\pi}\int_0^\pi \psi(t)\cos nt\,dt + J_1 + J_2 + J_3,$$

where $\rho = \frac{2m+1}{n}\cdot\frac{\pi}{2}$, $2m+1<n$, m being a positive integer and independent of n.

Since $\psi(t)$ is integrable in $(0, \pi)$, by Riemann-Lebesgue theorem, we have

$$\int_0^\pi \psi(t)\cos nt\,dt = 0\ (1) \quad \ldots \quad \ldots \quad (2{\cdot}2)$$

* Gupta,[1] perhaps through oversight, did not mention the condition of absolute continuity of (t) in the statement of his theorem although he made use of this property.

Integrating by parts we get

$$J_1 = -\frac{1}{4\pi}\Psi(\rho)\cot^2\frac{\rho}{2} + \frac{1}{4\pi}\int_0^\rho \Psi(t)\cot^2\frac{t}{2}\, n\sin nt\, dt$$

$$-\frac{1}{4\pi}\int_0^\rho \Psi(t)\cot\frac{t}{2}\operatorname{cosec}^2\frac{t}{2}\, dt + \frac{1}{4\pi}\int_0^\rho \Psi(t)\cot\frac{t}{2}\operatorname{cosec}^2\frac{t}{2}\cos nt\, dt$$

$$= -\frac{1}{4\pi}\Psi(\rho)\cot^2\frac{\rho}{2} + J'_1 + J'_2 + J'_3.$$

Since $\sin nt$ is of bounded variation in $(0, t)$, the total variation being equal to

$$\int_0^t \left| d\sin nt \right| \leqq nt,$$

we may put $\sin nt = P(t) - Q(t)$, where $P(t)$ and $Q(t)$ are positive, monotonic, and non-decreasing functions such that $|P(t)| < nt$ and $|Q(t)| < nt$.

Thus $n\sin t$. $P(t)$ and $n\sin t$. $Q(t)$ are both positive monotonic and increasing functions each being less than n^2t^2.

Now,

$$J'_1 = \frac{1}{8\pi}\int_0^\rho \Psi(t)\cot\frac{t}{2}\operatorname{cosec}^2\frac{t}{2}.\sin t\, n\sin nt\, dt$$

$$= \frac{1}{8\pi}\int_0^\rho \Psi(t)\cot\frac{t}{2}\operatorname{cosec}^2\frac{t}{2}\, n\sin t\, P(t)\, dt$$

$$-\frac{1}{8\pi}\int_0^\rho \Psi(t)\cot\frac{t}{2}\operatorname{cosec}^2\frac{t}{2}\, n\sin t\, Q(t)\, dt$$

$$\equiv L_1 + L_2,$$

where

$$L_1 \leqq n^2\rho^2\int_\xi^\rho \Psi(t)\cot\frac{t}{2}\operatorname{cosec}^2\frac{t}{2}\, dt,\ 0 \leqq \xi \leqq \rho$$

$$= \left[\frac{2m+1}{2}\pi\right]^2\int_\xi^\rho \Psi(t)\cot\frac{t}{2}\operatorname{cosec}^2\frac{t}{2}\, dt = 0(1),$$

when n tends to ∞ and consequently ρ and ξ tend to 0.

Similarly

$$L_2 = 0(1).$$

Therefore

$$J'_1 = 0(1).$$

In a similar manner by expressing cos nt in (o, ρ) as a difference of two monotonic, positive, and non-increasing functions we may prove that

$$J'_3 = 0\,(1).$$

Therefore

$$J_1 = 0(1) - \frac{1}{4\pi}\Psi(\rho)\cot^2\frac{\rho}{2} - \frac{1}{4\pi}\int_0^{\rho}\Psi(t)\cot\frac{t}{2}\operatorname{cosec}^2\frac{t}{2}\,dt \quad \ldots \quad \ldots \quad (2{\cdot}3)$$

Integrating by parts we have

$$J_2 = \frac{1}{4\pi}\Psi(\rho)\cot^2\frac{\rho}{2} - \frac{1}{4\pi}\int_{\rho}^{\pi}\Psi(t)\cot\frac{t}{2}\operatorname{cosec}^2\frac{t}{2}\,dt \quad \ldots \quad \ldots \quad (2{\cdot}4)$$

Now,

$$J_3 = \frac{1}{4\pi}\int_{\rho}^{\pi}\psi(t)\cot^2\frac{t}{2}\cos nt\,dt$$

$$= \frac{1}{4\pi}\int_{\rho}^{\pi}\psi(t)\left[\cot^2\frac{t}{2} - \frac{4}{t^2}\right]\cos nt\;dt + \frac{1}{\pi}\int_{\rho}^{\pi}\psi(t)\;\frac{\cos nt}{t^2}\,dt$$

$$= 0(1) + \frac{1}{\pi}\int_{\rho}^{\pi}\psi(t)\;\frac{\cos nt}{t^2}\;dt, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2{\cdot}5)$$

since $\psi(t)\left[\cot^2\frac{t}{2} - \frac{4}{t^2}\right]$ is integrable in $(0, \pi)$.

Hence from (2·2), (2·3), (2·4) and (2·5) we have

$$-I_1 + I_2 = -\frac{1}{4\pi}\int_0^{\pi}\psi(t)\,dt - \frac{1}{4\pi}\int_0^{\pi}\Psi(t)\cot\frac{t}{2}\operatorname{cosec}^2\frac{t}{2}\,dt$$

$$+ \frac{1}{\pi}\int_{\rho}^{\pi}\psi(t)\,\frac{\cos nt}{t^2}\,dt + 0\,(1) \quad \ldots \quad \ldots \quad (2{\cdot}6)$$

Now $\psi(t)$ being absolutely continuous, let

$$\psi(t) = \int_0^t h(t)\,dt.$$

On integrating by parts, we get

$$I_3 = -\frac{1}{2\pi}\int_0^{\pi}h(t)\sin nt\,dt$$

$$= 0(1), \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2{\cdot}7)$$

since $h(t)$ is integrable in $(0, \pi)$.

Again, integrating by parts we get,

$$I_4 = -\frac{1}{2\pi}\left[\psi(t)\cot\frac{t}{2}\cos nt\right]_o^\pi + \frac{1}{2\pi}\int_o^\pi \frac{d}{dt}\left[\psi(t)\cot\frac{t}{2}\right]\cos nt\,dt$$

$$= 0\,(1) + L,$$

since $\frac{\psi(t)}{t} \to 0$ as $t \to 0$.

Now,

$$L = \frac{1}{2\pi}\int_o^\pi h(t)\cot\frac{t}{2}\cos nt\,dt - \frac{1}{4\pi}\int_o^\pi \psi(t)\operatorname{cosec}^2\frac{t}{2}\cos nt\,dt$$

$$= \frac{1}{2\pi}\int_o^\pi h(t)\left[\cot\frac{t}{2} - \frac{2}{t}\right]\cos nt\,dt - \frac{1}{4\pi}\int_o^\pi \psi(t)\left[\operatorname{cosec}^2\frac{t}{2} - \frac{4}{t^2}\right]\cos nt\,dt$$

$$+ \frac{1}{\pi}\int_o^\rho \left[\frac{h(t)}{t} - \frac{\psi(t)}{t^2}\right]\cos nt\,dt + \frac{1}{\pi}\int_\rho^\pi \frac{h(t)}{t}\cos nt\,dt -$$

$$\frac{1}{\pi}\int_\rho^\pi \psi(t)\frac{\cos nt}{t^2}\,dt$$

$$= 0\,(1) + 0\,(1) + 0\,(1) + \frac{1}{\pi}\int_\rho^\pi \frac{h(t)}{t}\cos nt\,dt - \frac{1}{\pi}\int_\rho^\pi \psi(t)\frac{\cos nt}{t^2}\,dt,$$

since each of the following three functions

$$h(t)\left[\cot\frac{t}{2} - \frac{2}{t}\right],\quad \psi(t)\left[\operatorname{cosec}^2\frac{t}{2} - \frac{4}{t^2}\right],\ \text{and}$$

$$\frac{d}{dt}\left[\frac{\psi(t)}{t}\right] = \frac{h(t)}{t} - \frac{\psi(t)}{t^2}$$

is integrable in $(0, \pi)$.

Hence $$I_4 = 0\,(1) + \frac{1}{\pi}\int_\rho^\pi h(t)\frac{\cos nt}{t}\,dt - \frac{1}{\pi}\int_\rho^\pi \psi(t)\frac{\cos nt}{t^2}\,dt. \qquad \ldots\ (2{\cdot}8)$$

Therefore from (2·6), (2·7) and (2·8) we have

$$\lim_{n\to\infty} S_n = -\frac{1}{4\pi}\int_o^\pi \psi(t)\,dt - \frac{1}{4\pi}\int_o^\pi \Psi(t)\cot\frac{t}{2}\operatorname{cosec}^2\frac{t}{2}\,dt +$$

$$\lim_{n\to\infty}\frac{1}{\pi}\int_\rho^\pi \frac{h(t)}{t}\cos nt\,dt$$

$$= -\frac{1}{4\pi}\int_o^\pi \psi(t)\,dt - \frac{1}{4\pi}\int_o^\pi \Psi(t)\cot\frac{t}{2}\operatorname{cosec}^2\frac{t}{2}\,dt,$$

since $\frac{\psi(t)}{t} = \frac{1}{t}\int_o^t h\ (t)\ dt$ is continuous and of bounded variation and tends to 0 with t, we have by applying de la Vallée Poussin's criterion,

$$\lim_{n\to\infty} \int_\rho^\pi \frac{h\ (t)}{t} \cos nt\ dt = 0.$$

This completes the proof of the theorem.

3. In order to show that the theorem proved above is more general than that of Gupta, we shall require the following two Lemmas :—

Lemma 1. When the integral

$$\int_o^\delta \frac{\Psi\ (t)}{t^3} dt \qquad \ldots \qquad \ldots \qquad \ldots \quad (3{\cdot}1)$$

exists, the integral

$$\int_o^\delta \frac{\psi\ (t)}{t^2}\ dt \qquad \ldots \qquad \ldots \qquad \ldots \quad (3{\cdot}2)$$

will not exist unless $\lim_{t\to o} \frac{\Psi\ (t)}{t^2}$ *exists.*

If $\epsilon > 0$, we have on integrating by parts

$$\int_\epsilon^\delta \frac{\psi\ (t)}{t^2}\ dt = \frac{\Psi\ (\delta)}{\delta^2} - \frac{\Psi\ (\epsilon)}{\epsilon^2} + 2\int_\epsilon^\delta \frac{\Psi\ (t)}{t^3}\ dt \quad \ldots \qquad \ldots \qquad \ldots \quad (3{\cdot}3)$$

From (3·3) it is evident that unless $\lim_{t\to o} \Psi\ (t)/t^2$ exists, (3·2) will not exist.

Lemma 2. When the integral (3·2) *exists the integral* (3·1) *will always exist.*

It has been shown by Prasad[2] that

$$\mathrm{F}\ (x) = \int_o^x f(t)\ dt = 0\ (x^\delta),$$

where $\delta > 0$, provided

$$\int_o^x \frac{f\ (t)}{t^\delta}\ dt$$

exists. It follows, therefore, that whenever (3·2) exists,

$$\lim_{t\to o} \frac{\Psi\ (t)}{t^2} = 0.$$

Hence from (3·3) the lemma follows.

4. We have

$$\int_0^\pi \psi(t)\,\mathrm{cosec}^2\frac{t}{2}\,dt=\int_0^\pi \psi(t)\left\{\mathrm{cosec}^2\frac{t}{2}-\frac{1}{(t/2)^2}\right\}dt+4\int_0^\pi\frac{\psi(t)}{t^2}\,dt$$

$$\equiv A_1+A_2$$

Since $\left\{\mathrm{cosec}^2\frac{t}{2}-\frac{1}{(t/2)^2}\right\}$ is bounded and measurable, the integral A_1 will always exist. Hence the existence of (1·3) is equivalent to the existence of

$$\int_0^\pi\frac{\psi(t)}{t^2}\,dt.$$

Similarly it may be shown that the existence of (1·4) is equivalent to the existence of

$$\int_0^\pi\frac{\Psi(t)}{t^3}\,dt.$$

In view of the lemmas (1) and (2) and 2, it is evident that the theorem of this paper is more general than that of Mr. Gupta.

My thanks are due to Dr. B. N. Prasad for his keen interest in the preparation of this paper.

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DIFFERENTIATION OF A DEFINITE INTEGRAL WITH RESPECT TO A PARAMETER IN CERTAIN CASES WHEN LEIBNITZ'S RULE IS NOT APPLICABLE

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Communicated by Dr. Lakshmi Narayan

Received May 25, 1936

SUMMARY

The author, in this paper, has given rules for differentiation of a definite integral with respect to a parameter in two cases where Leibnitz's rule is not applicable. In the first case, he has considered those functions for which Leibnitz's rule is not applicable owing to the non-existence of the differential coefficient of these functions with respect to the parameter for certain values of the variable; and in the second case functions taken are of such a nature that the necessary and sufficient condition for the validity of Leibnitz's rule is not satisfied, although their differential coefficients with respect to the parameter exist everywhere in the domain.

1. In a previous paper [1] I investigated the necessary and sufficient condition for the validity of Leibnitz's rule for the differentiation of a definite integral with respect to a parameter. In this paper, which is in the nature of a supplement to the above, I will give rules (together with the conditions under which they hold) for the differentiation of a definite integral with respect to a parameter in two cases when Leibnitz's rule is not applicable.

Hobson [2] says "the problem has also been considered, of obtaining the differential coefficient when Leibnitz's rule is not applicable" without giving any reference as to the place and time of its publication. However, so far as my knowledge goes it has not been published anywhere.

2. Let a function $f(x, y)$ of the variables x and y be defined in the interval $a \leq x \leq b$; $c \leq y \leq d$ and satisfying the following conditions:—

(i) It is bounded and integrable (R) with respect to x in (a, b)

(ii) The differential coefficient

$$\frac{d}{dy} f(x, y)$$

exists for all values of y in (c, d) except possibly at the end points where there are only one-sided derivatives and for every value of x in the interval (a, b) except for a set of values of measure zero :

and

(iii) $\frac{d}{dy}\int_a^b f(x, y)\, dx$ and $\int_a^b \left\{ D_y(x, y) \right\} dx$

exist for all values of y in (c, d), where $D_y f(x, y)$ stands for any one of the four derivatives of $f(x, y)$ with respect to y.

Under the above assumptions, the rule for the differentiation of $\int_a^b f(x, y)\, dx$ with respect to y is as follows—

$$\frac{d}{dy}\int_a^b f(x, y)\, dx = \int_a^b \left\{ D_y f(x, y) \right\} dx.$$

and the necessary and sufficient condition for its validity is that corresponding to an arbitrarily chosen positive number ε and to another arbitrarily chosen positive number η a number h_1 such that $|h_1| < \eta$ can be found and also a positive number α such that

$$\left[\sum_{r=1}^{\frac{1}{m}} \frac{(b-a)\, m}{h_1} \left\{ f(a+\overline{b-a}\, rm,\ y+h_1) - f(a+\overline{b-a}\, rm,\ y) \right\} \right]$$

lies between

$$\left[\sum_{r=1}^{\frac{1}{m}} (b-a)\, m \left\{ \overline{D}_y f(a+\overline{b-a}\, r\, m,\ y) \right\} \right] + \varepsilon$$

and $$\left[\sum_{r=1}^{\frac{1}{m}} (b-a)\, m \left\{ \underline{D}_y f(a+\overline{b-a}\ rm,\ y) \right\} \right] - \varepsilon$$

for all values of m in $(0, \alpha)$ which are reciprocal to a positive integer, where $\overline{D}_y$ and $\underline{D}_y$ are the upper and lower derivatives both on the right or both on the left.

Proof :—Divide the interval (a, b) into n equal parts, then since

$\int_a^b f(x,y)\, dx$ exists as an R-integral

$$\frac{d}{dy}\int_a^b f(x, y)\, dx = \lim_{h\to 0} \lim_{m\to 0} \left[\sum_{r=1}^{\frac{1}{m}} \frac{(b-a)\, m}{h} \left\{ f(a+\overline{b-a}\ rm,\ y+h) - f(a+\overline{b-a}\ rm,\ y) \right\} \right]^{4}$$

Now by applying a theorem on repeated limits given by Hobson, it can be shown, as in my previous paper, [4] that the necessary and sufficient conditions in order that

$$\operatorname*{Lim}_{h\to o}\operatorname*{Lim}_{m\to o}\left[\sum_{r=1}^{\overline{\frac{1}{m}}}\frac{(b-a)\,m}{h}\left\{f\,(a+\overline{b-a}\,rm,\,y+h)-f(a+\overline{b-a}\,rm,\,y)\right\}\right]$$

$$=\operatorname*{Lim}_{m\to o}\operatorname*{Lim}_{h\to o}\left[\sum_{r=1}^{\overline{\frac{1}{m}}}\frac{(b-a)\,m}{h}\left\{f(a+\overline{b-a}\,rm,\,y+h)-f(a+\overline{b-a}\,rm,\,y)\right\}\right]$$

are

$$(a)\quad\overline{\operatorname*{Lim}_{h\to o}}\left[\sum_{r=1}^{\overline{\frac{1}{m}}}\frac{(b-a)\,m}{h}\left\{f(a+\overline{b-a}\,rm,\,y+h)-f(a+\overline{a-b}\,rm,\,y)\right\}\right]-$$

$$\underline{\operatorname*{Lim}_{h\to o}}\left[\sum_{r=1}^{\overline{\frac{1}{m}}}\frac{(b-a)\,m}{h}\left\{f(a+\overline{b-a}\,rm,\,y+h)-f(a+\overline{a-b}\,rm,\,y)\right\}\right]$$

converges to zero as $m\to o$ and that

(*b*) corresponding to an arbitrarily chosen positive number ε and to an arbitrarily chosen number η a number h_1 such that $|\,h_1\,|<\eta$ can be found and also a positive number α such that

$$\left[\sum_{r=1}^{\overline{\frac{1}{m}}}\frac{(b-a)\,m}{h_1}\left\{f(a+\overline{b-a}\,rm,\,y+h_1)-f(a+\overline{b-a}\,rm,\,y)\right\}\right]$$

lies between

$$\overline{\operatorname*{Lim}_{h\to o}}\left[\sum_{r=1}^{\overline{\frac{1}{m}}}\frac{(b-a)\,m}{h}\left\{f(a+\overline{b-a}\,rm,\,y+h)-f(a+\overline{b-a}\,rm,\,y)\right\}\right]$$

and

$$\underline{\operatorname*{Lim}_{h\to o}}\left[\sum_{r=1}^{\overline{\frac{1}{m}}}\frac{(b-a)\,m}{h}\left\{f(a+\overline{b-a}\,rm,\,y+h)-f(a+\overline{b-a}\,rm,\,y)\right\}\right]$$

for every value of m in the interval $(O,\,\alpha)$ which is the reciprocal of a positive integer.

The condition (a) above can be written as

$$\int_a^b \left\{ \overline{D}_y f(x, y) \right\} dx - \int_a^b \left\{ \underline{D}_y f(x, y) \right\} dx = 0$$

where $\overline{D}_y f(x, y)$ and $\underline{D}_y f(x, y)$ are the upper and lower derivatives of $f(x, y)$ with respect to y both on the right or both on the left. This condition is already satisfied as

$$\overline{D}_y f(x, y) = \underline{D}_y f(x, y)$$

except for a set of values of x of measure zero. The condition (b) alone is, therefore, the required necessary and sufficient condition, which can be written in the form given above.

3. Let $f(x, y)$ be a bounded function of x and y $(a \leq x \leq b\,;\, c \leq x \leq d)$ integrable (R) with respect to x then if

$$\frac{d}{dy} \int_a^b f(x, y)\, dx = \int_a^b \left\{ D_y f(x, y) \right\} dx$$

for all values of y $(\neq \alpha)$ in (c, d), α being such that $(c \leq \alpha \leq d)$, $\{D_y f(x, y)$ being any of the four derivatives of $f(x, y)$ with respect to $y\}$, and

$$\lim_{y \to \alpha} \int_a^b \left\{ D_y f(x, y) \right\} dx$$

exists then

$$\frac{d}{dy} \int_a^b f(x, y)\, dx \text{ at } y = \alpha$$

will exist and have the same value as

$$\lim_{y \to \alpha} \int_a^b \left\{ D_y f(x, y) \right\} dx.$$

Proof—

$$\because \lim_{y \to \alpha} \int_a^b \left\{ D_y f(x, y) \right\} dx$$

exists and also

$$\because \int_a^b \left\{ D_y f(x, y) \right\} dx = \frac{d}{dy} \int_a^b f(x, y)\, dx \text{ everywhere except at } y = \alpha.$$

$$\therefore \lim_{y \to \alpha} \frac{d}{dy} \int_a^b f(x, y)\, dx \text{ also exists,}$$

i.e.

$$\lim_{y \to \alpha} \frac{d}{dy} \int_a^b f(x, y)\, dx \text{ on the right}$$

and

$$\lim_{y \to \alpha} \frac{d}{dy} \int_a^b f(x, y)\, dx \text{ on the left,}$$

both exist and are equal.

Therefore the derivatives of $\int_a^b f(x, y)\,dx$ with respect to y at α exist both on the right and on the left and are equal[3]

i.e. $\frac{d}{dy}\int_a^b f(x, y)\,dx$ exists at $y=\alpha$

again $\because \operatorname{Lim}_{y\to\alpha} \frac{d}{dy}\int_a^b f(x, y)\,dx$ exists

therefore $\frac{d}{dy}\int_a^b f(x, y)\,dx$ can have no discontinuity of the second kind. Also, being a differential coefficient, it cannot have a discontinuity of the first kind. Hence it is continuous.

$$\therefore \frac{d}{dy}\int_a^b f(x, y)\,dx = \operatorname{Lim}_{y\to\alpha} \frac{d}{dy}\int_a^b f(x, y)\,dx \text{ at } y=\alpha.$$

$$= \operatorname{Lim}_{y\to\alpha} \int_a^b \left\{ \mathrm{D}_y f(x, y) \right\} dx.$$

4. *Illustrations*:—

(*a*) Let $f(x, y) = y^{1+|x-\frac{1}{2}|} \sin\frac{1}{y}$ when $y \neq 0$

$= 0$ when $y=0$

the function $f(x, y)$ at $y=0$ satisfies the condition given in Art. 2, *i.e.* for given positive numbers e and η a number h_1 ($|h_1| < \eta$) and also α can

be found such that $m+e > \left[\sum_{r=1}^{\frac{1}{m}} \frac{m}{h_1}\left\{ {h_1}^{1+|rm-\frac{1}{2}|} \sin\frac{1}{h_1} \right\}\right] > -(m+e)$,

for all values of m in $(0, \alpha)$

$$\text{for } \left[\sum_{r=1}^{\frac{1}{m}} \frac{m}{h_1}\left\{ {h_1}^{1+|rm-\frac{1}{2}|} \sin\frac{1}{h_1} \right\}\right] = \left[\sum_{r=1}^{\frac{1}{m}} m{h_1}^{|rm-\frac{1}{2}|} \sin\frac{1}{h_1}\right]$$

$$= m \sin\frac{1}{h_1}\Big[{h_1}^{|m-\frac{1}{2}|} + {h_1}^{|2m-\frac{1}{2}|} + \ldots + {h_1}^{\left|\left(\frac{1}{m}-2\right)m-\frac{1}{2}\right|} + {h_1}^{\left|\left(\frac{1}{m}-1\right)m-\frac{1}{2}\right|} + {h_1}^{\frac{1}{2}} \Big]$$

$$= m \sin \frac{1}{h_1} \left[2 \sum_{r=1}^{\frac{1}{2m}-1} h_1^{\,|rm-\frac{1}{2}|} + h_1^{\frac{1}{2}} + 1 \right] \text{ or}$$

$$m \sin \frac{1}{h_1} \left[2 \sum_{r=1}^{\frac{1}{2m}-\frac{1}{2}} h_1^{\,|rm-\frac{1}{2}|} + h_1 \right]$$

according as $\frac{1}{m}$ is even or odd.

$$= m \sin \frac{1}{h_1} \left[2 \left(\frac{h_1^{\frac{1}{2}-m} - 1}{1 - h_1^{\,m}} \right) + h_1^{\frac{1}{2}} + 1 \right] \text{ or}$$

$$m \sin \frac{1}{h_1} \left[2 \left(\frac{h_1^{\frac{1}{2}-m} - h_1^{\,\frac{m}{2}}}{1 - h_1^{\,m}} \right) + h_1^{\frac{1}{2}} \right]$$

according as $\frac{1}{m}$ is even or odd.

Now if we take m to be such a function of h_1 that $\lim_{h_1 \to o} m = 0$ and $\lim_{h_1 \to o} (m \log h_1) = 0$ (this is quite possible for example $m = \frac{1}{(\log h_1)^2}$) then the limits of the above two expressions as $h_1 \to o$ become

$$\lim_{h_1 \to o} m \sin \frac{1}{h_1} \left[2 \left\{ \frac{h_1^{\frac{1}{2}} (1 - m \log h_1 + A) - 1}{m \log h_1 - A} \right\} + h_1^{\frac{1}{2}} + 1 \right] \text{ or}$$

$$\lim_{h_1 \to o} m \sin \frac{1}{h_1} \left[2 \left\{ \frac{h_1^{\frac{1}{2}} (1 - m \log h_1 + A) - 1 + \frac{m}{2} \log h_1 - B}{m \log h_1 - A} \right\} + h_1^{\frac{1}{2}} \right]$$

according as $\frac{1}{m}$ is even or odd

where A stands for $\frac{m^2 (\log h_1)^2}{\lfloor 2} - \frac{m^2 (\log h_1)^3}{\lfloor 3} + \ldots\ldots$

and B stands for $\frac{m^2 (\log h_1)^2}{4 \lfloor 2} - \frac{m^3 (\log h_1)^3}{8 \lfloor 3} + \ldots\ldots$

$= 0$ whether $\frac{1}{m}$ is even or odd.

Hence $\frac{d}{dy}\int_o^1 f(x, y)dx = \int_o^1 \left\{ D_y\ f(x, y) \right\} dx$

as a matter of fact both sides of the above equation are zero

(*b*) Let $f(x, y) = \phi_1(x, y) + \phi_2(x, y)$

where $\phi_1(x, y) = y^{1+\left|x-\frac{1}{2}\right|}\sin\frac{1}{y}$ when $y \neq 0$

$= 0$ when $y = 0$

and $\phi_2(x, y) = \left(1+\frac{y}{x}\right)e^{-\frac{y}{x}}$ when $x \neq 0$

$= 0$ when $x = 0$

The above function does not satisfy the condition given in Art. 2 as

$$\sum_{r=1}^{\frac{1}{m}} \frac{m}{h_1} \left\{ \phi_2(rm_1\ h_1) - \phi_2(rm_1\ 0) \right\}$$

cannot be made arbitrarily small by taking m and h_1 small enough.

Hence $\frac{d}{dy}\int_o^1 f(x, y)\,dx \neq \int_o^1 \left\{ D_y\ f(x, y) \right\} dx$

as a matter of fact by actual calculation

$$\frac{d}{dy}\int_o^1 f(x, y)\,dx = -1 \text{ and } \int_o^1 \left\{ D_y\ f(x, y) \right\} dx = 0.$$

(*c*) Let $f(x, y) = \sin\left(4\tan^{-1}\frac{y}{x}\right) - \frac{4xy}{x^2+y^2}\cos\left(4\tan^{-1}\frac{y}{x}\right)$

Then since $\frac{d}{dy}\int_o^X f(x, y)\,dx = \int_o^X \left\{ \frac{d}{dy} f(x, y) \right\} dx$ at $y = 0$

both being equal to $\frac{4X^2}{X^2+y^2}\cos\left(4\tan^{-1}\frac{y}{X}\right)$

and $\lim_{y\to o}\int_o^X \left\{ \frac{d}{dy} f(x, y) \right\} dx$ exists

$$\therefore \frac{d}{dy}\int_o^X f(x, y)\,dx = \lim_{y\to o}\int_o^X \left\{ \frac{d}{dy} f(x, y) \right\} dx \text{ at } y = 0$$

In fact both of them are equal to 4.

[This example has been given by Harnack[1] and Hobson[2] to illustrate the inapplicability of Leibnitz's rule at $y=0$. As shown above, I have indicated a method of finding the differential coefficient in such cases.]

(*d*) The function

$$f(x, y) = \left(1+\frac{y}{x}\right) e^{-\frac{y}{x}} \qquad \text{when} \quad x=0$$

$$= 0 \qquad \text{when} \quad x=0$$

also satisfies the condition given in Art. 3

$$\therefore \frac{d}{dy}\int_0^1 f(x, y)\,dx = \operatorname*{Lim}_{y\to 0} \int_0^1 \left\{\frac{d}{dy} f(x, y)\right\} dx \quad \text{at} \quad y=0$$

[This example was given by the author in a previous paper[4] to illustrate the inapplicability of Leibnitz's rule.]

My thanks are due to Dr. Lakshmi Narayan under whose guidance this investigation has been carried out.

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CHEMICAL EXAMINATION OF THE SEEDS OF *PHYSALIS PERUVIANA* OR CAPE GOOSEBERRY, PART II

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Communicated by Dr. S. Dutt.

Received July 1, 1936

SUMMARY

The seeds of *Physalis peruviana* on extraction with benzene yielded 6·23% of an oil of light yellow colour, which is a glyceride of oleic, linolic (unsaturated) and palmitic and stearic (saturated) acids. The unsaponifiable matter consists of a phytosterol $C_{28}H_{44}O_3$. The alcoholic extract showed the presence of saponins and glucose only.

Physalis peruviana (N. O. Solanaceae) called cape gooseberry in English and Makoi in Hindustani, is cultivated in India and hardly differs from *Physalis minima* Linn. except in its large size and more oblong berry. It affords an excellent fruit and is much cultivated in this country. The berries are globular, with slight elongation, about $\frac{3}{4}''$ in length and breadth and having numerous pale yellow tiny seeds embedded in juicy pulp. The seeds constitute 4·5-5·0 per cent of the weight of the husk-free fresh ripe berries. The seeds have been submitted to a detailed chemical examination and, as a result, it is shown that the seeds contain 6·28 % of a pale yellow semi-drying oil which has been worked up in detail as described in the experimental part of the paper. No alkaloid or glucoside could be detected but the presence of an amorphous saponin has been established.

Experimental

The dried seeds of the berries were collected as described in the part I and were ground with great difficulty owing to their small size and hardness. 50 gms. of the crushed seeds were successively extracted with benzene and alcohol in a Soxhlet's apparatus. On subsequent evaporation of the solvent and drying of the extract at 100° to a constant weight the following results were obtained :—

Benzene Extract :—Light yellow oil. Yield 6·23 %.

Alcoholic Extract :—Tasteless light brownish syrupy mass containing some oily or fatty matter and giving a faint greenish coloration with ferric chloride. Yield 3·58%.

The seeds when completely burnt in a porcelain dish left 7·46% of greyish white ash. The ash contained the following positive and negative radicals :—Potassium, sodium, iron, calcium, aluminium, phosphate, silica and chloride.

For complete analysis 2·8 kgs. of the crushed seeds were exhaustively extracted with benzene in a 5 litre extraction flask. The combined benzene extracts on complete distillation of the solvent gave 182 gms. of a pale yellow oil of very thin consistency. The oil-free powder after removal of the traces of benzene was completely extracted with rectified spirit. The combined extract which was tasteless and light brownish yellow in colour was concentrated and then allowed to stand for several days when no crystalline substance separated. It was then repeatedly extracted with hot benzene in order to free it from oil. The alcoholic extract gave a very light green coloration with alcoholic ferric chloride and no precipitate with alcoholic lead acetate. On dilution with water it gave a little flocculent white precipitate which was filtered with great difficulty at the pump and after drying in vacuum desiccator melted at 87°—89°C and was brownish in colour. The amount, being only 0·3 gm., could not be obtained pure.

The diluted alcoholic extract gave no precipitate with lead acetate but a pale yellow precipitate was obtained on addition of a slight excess of basic lead acetate. The precipitate was filtered on the filter pump well washed with hot water and then decomposed in alcoholic suspension with sulphurated hydrogen. The alcoholic filtrate and washings from the resulting lead sulphide deposited globules but no crystalline substance. It gave definite tests for saponins such as frothing on shaking with water, a blue coloration with a solution of potassium ferricyanide and ferric chloride, reduction of mercuric chloride to mercurous chloride, reddish violet coloration with concentrated sulphuric acid.

The filtrate and washings from the basic lead lake was concentrated on the water bath after removal of lead as lead sulphide but did not deposit any crystalline substance even on keeping for several days. It gave an osazone which after crystallisation from alcohol melted at 203°C and was identified as phenyl glucosazone thus proving the presence of glucose.

The crude oil was digested with animal charcoal and filtered through a hot funnel. The purified oil was freed from traces of benzene by heating over water bath and finally by keeping in vacuum desiccator. It is slightly optically active having a small rotation in chloroform $[\alpha]_D^{20} = -38°$. The fatty acids obtained after saponification of the oil are optically inactive, which shows that the rotation is due to the sterol present. The oil burns with a non-sooty flame, and gives positive colour reactions for sterols. The oil is tasteless, semi-drying and free from nitrogen and sulphur. The physical and chemical constants of the oil are $(R.D.)_{27}^{27} = 0·881$; remains clear up to − 10° but becomes thick; acid value = 39·06; saponification value = 179·6; acetyl value = 41·58; unsaponifiable matter = 0·4%; Hehner's value = 93·4; iodine value = 120·5.

150 gms. of the oil were saponified with excess of alcoholic caustic potash and after complete removal of alcohol the resulting soap was freed from alcohol and

then repeatedly extracted with ether in order to remove the unsaponifiable matter. From the resulting soap mixed fatty acids were recovered. The pure mixed fatty acids have $(R.D.)_{23}^{23}=0{\cdot}8826$; neutralisation value=195·8; mean molecular weight =286·1; iodine value=122·45.

100·00 gms. of mixed fatty acids were separated into saturated and unsaturated acids by the Twitchell's[2] Lead-salt-alcohol method as this leads to more quantitative separation than by the lead-salt-ether method (Table I). But during the separation of unsaturated acids, a small amount of resinous acids insoluble in petroleum ether but soluble in ether also separated.

Table I

Acids	% in mixed acids	% in the oil	Iodine value	Mean molecular weight
Saturated ...	13·40	12·52	2·7	286·2
Unsaturated ...	86·60	80·59	134·2	278·7

Elaidin test for the liquid acids :--The liquid acids gave a positive test for elaidic acid.

Oxidation of unsaturated Acids with Potassium permanganate :—10 gms. of the acids were dissolved in aqueous caustic potash and oxidised with a solution of potassium permanganate at room temperature with constant stirring and after the reaction a current of SO_2 was passed to dissolve the precipitated manganese dioxide. The insoluble oxidation product was filtered from it, by successive extraction with ether and boiling water, dihydroxy stearic acid melting at 131-32°C and tetrahydroxy stearic acid (sativic acid) melting at 164-65°C were isolated showing thereby the presence of oleic and linolic acid; hexa-hydroxy stearic acid was entirely absent showing the absence of linolic acid.

The constituents of the unsaturated acids were quantitatively estimated by means of their bromine addition products as recommended by Jamieson and Baughmann.[1] The hexa-bromo-derivative of linolenic acid is insoluble in cold ether, since no precipitate insoluble in ether was formed, the absence of linolenic acid was confirmed. The ether soluble portion was dissolved in petroleum ether and cooled when crystals of linolic tetra-bromide (m. p. 113°C-114°C) were separated showing the presence of linolic acid. The residue was evaporated to dryness and the bromine content estimated. Table III contains the data regarding the analysis of the bromo-derivatives.

The iodine value of the mixture of unsaturated acids was found to be 134·2. The proportion of linolic acid and oleic acid was calculated by the help of the following equations :—

$$X + Y = 100 \quad \ldots \quad (1)$$

$$90{\cdot}07X + 181{\cdot}14Y = 100I \quad \ldots \quad (2)$$

where X and Y are percentages of oleic and linolic acids respectively and I the iodine value of unsaturated acids. The results are given in Table II.

Table II

	% in the unsaturated acids	% in the total fatty acids	% in the original oil
Oleic acid ...	51·55	44·64	41·69
Linolic acid ...	48·45	41·96	39·19

Table III

Wt. of the unsaturated acids taken	5·6920 gms.
Linolic tetrabromide insoluble in petrolium ether ...	3·3712 gms.
Residue (dibromide and tetrabromide)	6·5843 gms.
Bromine content of the residue	32·1%
Dibromide in the residue 72·5% or	4·7720 gms.
Tetrabromide in the residue 27·5% or	2·2780 gms.
Total tetrabromide found	5·6492 gms.
Linolic acid equivalent to tetrabromide	2·6360 gms. or 46·40%
Oleic acid equivalent to dibromide	3·0460 gms. or 53·60%

Table IV gives the percentage of oleic and linolic acids in the unsaturated acids calculated from the above data.

Table IV

	% in the unsaturated acids	% in the total fatty acids	% in the original oil
Oleic acid ...	53·60	46·42	42·37
Linolic acid ...	46·40	40·18	37·53

The theoretical iodine value of a mixture consisting of 53·6 % of oleic acid and 46·42 % of linolic acid is 132·29, which agrees fairly well with the observed iodine value of the unsaturated acids (134·2).

Saturated Acids :—The saturated acids separated by the lead-salt-alcohol method were freed from traces of liquid acids if any by pressing over porous plate and were yellowish white crystalline solid melting at 54-56°C. The pure methyl esters obtained from 10·08 gms. of mixed saturated acid were fractionated under reduced pressure, the boiling points and the pressures being recorded as shown in the Table V and the result of analysis and percentage of acids in the different fractions are given in Table VI.

Table V

Fraction	B. P.	Pressure in mm.	Wt. in gms.
1 ...	169—175°C ..	2 ...	4·4720 ...
2 ...	175—185°C ...	2 ...	2·3296 ...
3 ...	Residue ...	2 ...	2·2944 ...

Table VI

Fraction	Iodine value	Sap. value	Molecular wt.	Unsaturated acids gm.	%	Molecular wt. of esters of saturated acids	Palmitic acids gm.	%	Stearic acids gm.	%	Arachidic acids gm.	%
1	2·3	204·5	274·3	0·0692	1·55	270·9	4·0760	91·12	0·8744	1·96	...	...
2	3·5	195·1	287·6	0·0552	2·36	287·4	0·9184	35·13	1·2788	54·90	...	..
3	9·9	187·5	299·2	0·1528	6·66	299·4	...	...	1·9164	83·60	0·1116	4·42

The percentage of the various acids in the total saturated acids is given in Table VII.

Table VII

Acids	% found in the saturated acids	% in the original oil
Palmitic	52·38	6·56
Stearic	43·38	5·45
Arachidic	1·11	0·14
Unsaturated acids ...	2·97	0·37

Examination of the unsaponifiable matter :—The residue left on concentration of the ethereal extract of the soap was dissolved in 150 c.c. of distilled water and repeatedly extracted with ether. The combined ethereal extracts after a few washings with water was dried over sodium sulphate and ether completely distilled. The crystalline residue on repeated crystallisation from small quantity of rectified spirit was obtained as silky needles melting sharp at 131°C. It gave the various colour reactions of sterols and was optically active, $[\alpha]_D^{23} = -34{\cdot}04^\circ$ in chloroform (c=1·7885). A sample crystallised from chloroform and air dried was combusted. (Found: C=78·40; 78·42; H=10·52, 10·49; $C_{28}H_{44}O_3$ requires C=78·50 %; H=10·28%.) It is a new sterol and the formula given is provisional.

The Authors wish to express their indebtedness to Dr. S. Dutt., D.Sc., P.R.S., for his kind interest in the work.

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CHEMICAL EXAMINATION OF THE SEEDS OF ISABGHOL *PLANTAGO OVATA* FORSKS. PART II. (SUPPLEMENTARY NOTE)

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Communicated by Dr. S. Dutt

Received May 7, 1937

SUMMARY

The saturated acids in the oil from the seeds of Isabghol which are 12·43 % of the total fatty acids and 11·42 % in the original oil contain palmitic acid (32·77 %); stearic acid (60·37 %); and lignoceric acid (6·86 %).

In a previous paper on the "Chemical Examination of the seeds of Isabghol, *Plantago ovata* Forsks" the seeds were subjected to a systematic chemical examination. The investigations revealed the presence of a non-volatile oil, large amount of mucilaginous matter and a quantity of reducing sugars. The oil has been analysed. The composition of the saturated acids could not be determined then. As the investigation is not complete without the examination of the saturated acids, the present note is on the complete and systematic analysis of the saturated acids. The saturated acids from the oil contain palmitic, stearic and lignoceric acids.

Experimental

125 grams of the oil from the seeds were saponified in theu sual way with alcoholic caustic potash and the unsaponifiable matter was separated by ether from the soap solution. The fatty acids liberated from the soap were separated into the unsaturated liquid and saturated solid acids by the Twitchell's lead-salt-alcohol method. About 14 gs. of the saturated acids were obtained from the mixed acids.

Examination of the saturated acids :—The saturated acids separated as above, were freed from traces of liquid acids by pressing them over a porous plate. The acids thus obtained were perfectly solid yellowish white in colour and melted at about 54°. The iodine value was 2·08 and the mean molecular weight calculated from the saponification value (197·4) was 284·2. The mixture of the fatty acids was

then converted into the methyl ester by refluxing 12 grams of the acids with absolute methyl alcohol saturated with hydrochloric acid gas for over 16 hours. The methyl esters (12 gs.) were then fractionally distilled under reduced pressure. The saponification values were determined for different fractions and their mean molecular weights calculated. The free acids were liberated from different fractions and their melting points recorded. The last two fractions had a higher molecular weight and the melting points of the liberated acids probably point to the presence of lignoceric acid. The following tables contain the results of the analysis :—

Table I

Fractions	Boiling points and pressure	Weight of fraction	Saponification value	Mean Mol. Wts. of the esters	Melting points of liberated acids
1	165-75°/1·5 m.m.	2·72 gms.	203·2	276·1	62°-630°
2	175-180°/1·5 m.m.	2·24 gms.	198·4	282·8	65°
3	180-92°/1 m.m. rising up to 200°	1·32 gms.	194·2	289·6	67°-68°
4	200-208°/1 m.m.	3·24 gms.	184·6	304·0	74°-75°
5	Residue (above 210°)	2·48 gms.	176·4	318·1	76°-77°
		12·00			

The iodine value of all the fractions was below 1·5 and therefore no correction is made in the mean molecular weights of the ester.

Table II

Fractions	Esters			Acids			Acids in gms.		
	Palmitic	Stearic	Lignoceric	Palmitic	Stearic	Lignoceric	Palmitic	Stearic	Lignoceric
1	79·36%	20·64%	...	75·23%	18·78%	...	2·05	0·51	...
2	55·52%	44·48%	...	52·64%	40·48%	...	1·18	0·91	...
3	31·31%	68·69%	...	29·70%	62·52%	...	0·39	0·83	...
4	...	93·3%	6·7%	...	84·92%	6·45%	...	2·75	0·21
5	...	76·43%	23·57%	...	69·56%	22·71%	...	1·67	0·55
							3·62	6·67	0·76
							Total	11·05	

Table III

	Acids	In the saturated acids	In the mixed acids	In the original oil
1	Palmitic ...	32·77 %	4·07 %	3·74 %
2	Stearic ...	60·37 %	7·50 %	6·89 %
3	Lignoceric ...	6·86 %	0·86 %	0·79 %
			12·43 %	11·42 %

The author is highly grateful to Dr. S. Dutt of the Allahabad University for his valuable guidance and kind interest during the progress of the work and to the authorities of the Victoria College, Gwalior, for facilities given to him throughout the investigation.

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SIMILARITY BETWEEN NITROGEN FIXATION AND CARBON ASSIMILATION

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Received January 27, 1937

SUMMARY

1. Experimental results show that the nitrogen fixation per gram of carbon oxidised in sun or artificial light is much greater than that in the dark. This happens *in vitro*, in dishes with soils and in fields.

2. The nitrogen fixation when soil is mixed with energy-rich materials, is always greater in light than in the dark, although the Azotobacter count in light is much less than in the dark.

3. It appears that the energy required for nitrogen fixation can be supplied from the energy of the oxidation of energy materials and from sunlight or any other source of light.

4. In presence of glucose, the nitrogen fixation in the soil kept in the dark is highest at 35° but is much less than in light. At higher temperatures, the nitrogen fixation in the dark is much less.

5. Our results show that in nitrogen fixation sunlight or artificial light is used up as in carbon assimilation.

It is well known that most of the photochemical reactions, which have been extensively studied, *e.g.*, between chlorine and hydrogen, chlorine and carbon-monoxide, organic substances and chlorine and bromine, reaction between oxalate and iodine, oxalate and mercuric chloride, etc., are exothermal chemical reactions. The incident light just initiates or starts the chemical changes and may not be necessary for the progress of the reaction. As considerable amounts of energy are set free in these reactions, there is no proportionality between the number of molecules decomposed or formed and the number of quantum of light absorbed by the reacting systems and in the majority of these reactions Einstein's law of photochemical equivalence fails, as many molecules are formed or decomposed per quantum of light absorbed.

On the other hand, the photosynthesis of energy materials on the leaf surface according to the equation

$$6\,CO_2 + 6\,H_2O + 676\,Cal = C_6H_{12}O_6 + 6O_2$$

is the most important endothermal chemical reaction, which has been studied by chemists, agriculturists and plant physiologists for over hundred and fifty years. This reaction apparently cannot proceed in the absence of light, which supplies the energy required in this highly endothermal reaction.

For over twentyfive years in numerous publications from this laboratory, we have been investigating the initiation of exothermal reactions by the energy obtained from another exothermal reaction. Thus we have shown that the energy derived from the oxidation of an aqueous solution of oxalic acid by potassium permanganate, can form a large amount of calomel from a mixture of mercuric chloride and oxalic acid even in the dark. Similarly the oxidation of different organic acids by potassium permanganate, manganese dioxide, persulphuric acid, nitrous acid, iron salts and hydrogen peroxide and other oxidizing agents can induce or start several chemical reactions. Moreover, the oxidation of sodium sulphite or ferrous hydroxide or cerous hydroxide or manganous hydroxide, etc., by air leads to the oxidation of nickelous hydroxide or carbohydrates, fats, proteins, etc, even in the absence of light. The energy materials like carbohydrates, fats, and proteins have also been oxidised simply by passing air through their solutions or suspensions exposed to light. In all these cases exothermal reactions have been initiated either through the agency of light or the energy available from another exothermal chemical reaction.

Recently we have been successful in inducing an endothermal reaction $N_2+O_2+43{\cdot}2$ Cal$=2NO$ by the energy obtained from the oxidation of different carbohydrates by air. Thus Dhar and Mukerji [2, 6] have obtained four to six milligrams of nitrogen fixation per gram of canesugar or glucose oxidised (*e.g.*, about 12 milligrams of nitrogen fixed per gram of carbon oxidised) by passing sterile air through solutions of the carbohydrates containing ferrous hydroxide or cerous hydroxide or sterile soil in the complete absence of bacteria in the dark. Recently Dhar and Sundara Rao have obtained nitrogen fixation of 9 milligrams per gram of canesugar oxidised (*e.g.*, about 23 milligrams of nitrogen fixed per gram of carbon oxidized) in the sunlight by passing air through the solutions of the carbohydrate in contact with ferrous or cerous hydroxide. These experimental results show definitely that per gram of energy material oxidised, the nitrogen fixation in sunlight is practically double of that obtained in its absence.

We have carried on numerous experiments on nitrogen fixation when soil mixed with energy materials are exposed to light or kept in the dark. These experiments have been carried on with soils kept in dishes and with field soils. When 20 tons of molasses containing 60% carbohydrates are added per acre of normal soil exposed to air and sunlight which is turned over once every month, 493 lbs. of nitrogen are fixed. Using 40 tons, 605 lbs. of nitrogen are fixed, whilst with 3 tons and 10 tons of molasses per acre, the nitrogen fixations are 112 lbs. and 270 lbs. respectively. Carbohydrate manuring in tropical countries appears to be better than symbiotic nitrogen fixation, which adds 50—100 lbs. nitrogen per acre.

The following results on nitrogen fixation have been obtained with soils kept in light and in the dark.

Table I

Nitrogen fixed per gram of carbon oxidised

Experiments in dishes

Canesugar + Ca CO_3	(Exposed)	15·8	mgm.
Canesugar	„	14·7	„
Canesugar + Ca CO_3	(Dark)	10·5	„
Canesugar	„	10·2	„
Glucose + Ca CO_3	(Exposed)	12·5	„
Glucose	(Exposed)	12·5	„
Glucose + Ca CO_3	(Dark)	6·5	„
Glucose	(Dark)	6·5	„
Mannitol	(Exposed)	12·8	„
Mannitol	(Dark)	6·9	„
Starch	(Exposed)	7·58	„
Starch	(Dark)	3·59	„
Glycerol	(Exposed)	6·04	„
Glycerol	(Dark)	3·1	„
Cowdung	(Exposed)	28·3	„
Cowdung	(Dark)	13·6	„
	Field Trials		
Plot 6′ × 4′ containing 5 Kg. of molasses.	(Exposed)	22·9	„
	(Dark)	13·2	„
Plot 6′ × 4′ containing 10 Kg. of molasses.	(Exposed)	17·8	„
	(Dark)	10·06	„

Exactly similar results have been obtained with butter, hay and other energy materials.

The following results were obtained by exposing soil and canesugar to light from a 1000 watt filament lamp.

Table II

Experiments in dishes at 40°. Nitrogen fixed per gram of carbon oxidized

1% Canesugar	(Exposed)	33	milligrams
1% Canesugar	(Dark)	6·2	„
2% Canesugar	(Exposed)	23·0	„
2% Canesugar	(Dark)	6·8	„
3% Canesugar	(Exposed)	20	„
3% Canesugar	(Dark)	5·5	„
5% Canesugar	(Exposed)	14	„
5% Canesugar	(Dark)	5·4	„

The foregoing results definitely show that nitrogen fixation in sunlight or artificial light is much greater than that obtained in the dark per gram of the energy material oxidised. As the amount of energy available from the oxidation of one gram of energy material is the same both in the light and the dark, it is expected that the nitrogen fixation should be the same in light as in the dark, but the experimental results show that with all the energy materials, the nitrogen fixation for the same amount of energy output from the oxidation of the energy material, in light is much more than in the dark. Hence we are led to conclude that in nitrogen fixation, which is an endothermal reaction, the radiations of the sun or from other sources are actually utilised in effecting more nitrogen fixation as in photosynthesis or carbon assimilation in plants or in the formation of ozone by light absorption.

In a recent communication to "NATURE," Dhar and Mukerji[5] have reported that when soil is mixed with cowdung and exposed to air and light there is considerable nitrogen fixation. Our results show that with cowdung as an energy material the nitrogen fixation in the fields receiving sunlight is much greater than in the dark when compared per gram of carbon oxidized.

Dhar and Seshacharyulu have determined the numbers of Azotobacter in soils containing energy materials and the amount of nitrogen fixed in light and dark and they have shown that the number of Azotobacter per gram of soil kept in light is much smaller than in the soil kept in the dark although the nitrogen fixation in light is much greater than in the dark. The following tables show some typical results obtained in this direction.

Table III

1 kilogram of soil + 50 gms. starch. Dishes exposed to sunlight daily for 8 hours. (Temperature 36° - 48°)

Date of analysis	NH_3N	Total -N	Total -C	Azotobacter in millions per gram of dry soil	
10-3-1936	0·001%	0·042%	0·441%	7·2	
9-4-1936	0·001	0·042	.....	6·8	
30-4-1936	0·0018	0·0433	2·4425	8·2	
20-5-1936	0·0023	0·0442	2·37	9·1	
20-6-1936	0·0029	0·0451	2·2765	15·6	Nitric nitrogen varied from 0·0024 % to 0·00336 %
16-7-1936	0·0033	0·0461	2·1753	19·8	
28-7-1936	0·0037	0·0472	2·0932	28·2	
7-10-1936	0·0052	0·051	1·4411	38·6	
7-11-1936	0·0056	0·053	1·2592	31·2	
7-12-1936	0·0058	0·0538	1·0181	39·4	
2-1-1937	0·006	0·0555	0·7217	32·8	
6-2-1937	0·00464	0·056	0·5953	28·5	

Nitrogen fixed per gram of carbon oxidized in sunlight = 7·58 milligrams.

Dishes kept in the dark.
(Temperature 30° – 38°)

Date of analysis	NH_3N	Total -N	Total -C	Azotobacter millions per gram of dry soil	
12-3-1937	0·004	0·0547	0·5684	16·5	
11-4-1937	0·00356	0·053	0·5486	9·5	
10-3-1936	0·001 %	0·042	0·441	7·2	
6-4-1936	0·0010	0·042	... ...	8·1	Nitric nitrogen varied from 0·0024% to 0·00264 %
30-4-1936	0·0014	0·0420	2·5682	11·2	
20-5-1936	0·0015	0·0420	2·4761	18·5	
20-6-1936	0·0018	0·0433	2·4137	26·8	
16-7-1936	0·0021	0·0437	2·3352	46·0	
28-7-1936	0·0023	0·0442	2·2654	98·6	
7-10-1936	0·0031	0·0461	1·8286	205·0	
7-11-1936	0·0032	0·0461	1·7164	265·0	
7-12-1936	0·0032	0·0466	1·5382	305·0	
2-1-1937	0·0035	0·0472	1·3124	365·5	
6-2-1937	0·0032	0·0477	1·0538	380·0	
2-3-1937	0·003	0·0482	0·7854	385·0	
11-4-1937	0·0024	0·0482	0·5368	300·0	

Nitrogen fixed per gram of carbon oxidized in the dark = 3·13 milligrams.

From the foregoing observations it is clear that nitrogen fixation can take place in the dark when the soil is supplied with energy obtained from the oxidation of energy materials, like carbohydrates, pentosans, cellulose etc., but the nitrogen fixation is considerably increased when these systems also receive sunlight or artificial light, which is utilized in this process as in photosynthesis in the plant kingdom or in the formation of ozone by light absorption.

Recent experiments of Dhar and Seshacharyulu show that the nitrogen fixation in soil fed with 2 % glucose in the dark is 7·76 milligrams per gram of carbon oxidized at 35°. At higher or lower temperatures the nitrogen fixation in the dark is much less, whilst under identical conditions the nitrogen fixation in sunlight is 13·1 milligrams per gram of carbon oxidized kept at a temperature of 42°. Hence light and not increase of temperature is responsible for increased nitrogen fixation in light.

Many plant physiologists have reported that there is an intimate relationship between photosynthesis and respiration in plants. Thus Plester[10] has stated that while it is evident that there is no parallelism between chlorophyll-content and rate of photosynthesis, the quotient $\frac{\text{Respiration}}{\text{Photosynthesis}}$ for the light green varieties seems fairly constant, as is evident from the following results :—

Ptelea = 1·77, Catalpa = 1·72, Mirabilis = 2·0,
Ulmus = 2·0, Populus = 2·1.

Henrici,[8] Boysen – Jensen[1], Spoehr and McGee[11] and others have found that plants, which have a high rate of respiration, have also a high rate of carbohydrate formation.

Dhar and Atma Ram[3] have postulated that in nature the photosynthesis that is taking place in the plants by the absorption of sunlight is aided by the energy obtained in plant respiration, which goes on as long as the plant lives.

When solutions of organic substances are exposed to light and air they are oxidized photochemically to carbondioxide and water along with the formation of formaldehyde. The following results have been obtained by Dhar and Atma Ram :—

Table IV

Comparative experiments in the photosynthesis of formaldehyde *in vitro* from sodium salts of fatty acids, carbohydrates, proteins and potassium carbonate solutions exposed to sunlight in quartz vessels.

Temperature 35°, volume of solution exposed=25c. c. for 6 hours

System exposed	Amount of substance decomposed or photo-oxidised in gram moles.	Percentage of the substance oxidised or decomposed	Amount of formaldehyde formed in gram moles.	Ratio of amount of substance oxidised or decomposed to that of formaldehyde formed
M/5 $KHCO_3$	0·098	4·9	0·000001	9800
M/100 Sodium oleate.	0·00011	1·1	0·0000037	29
M/100 Sodium palmitate.	0·000091	0·9	0·000003	30
M/100 Sodium stearate.	0·000082	0·82	0·0000023	36
M/100 Cane-sugar.	0·00015	1·5	0·0000018	83
M/100 Glucose	0·00016	1·6	0·0000017	126
M/100 α-alanine	0·00021	2·1	0·0000018	117
M/100 Aspartic Acid.	0·00018	1·8	0·0000015	120

The foregoing results show that the amount of formaldehyde formed by exposing potassium bicarbonate solutions to sunlight in quartz vessels is smaller than the amount formed in the photo-oxidation of the organic substances. Moreover, when we compare the amounts of formaldehyde formed with the number of molecules of bicarbonate decomposed or the organic substance oxidised, a great difference is at

once observed. Although the salts of the fatty acids are oxidised to a smaller extent than the carbohydrates and the amino acids under comparable conditions, the amounts of formaldehyde produced are greater in the case of the salts of the fatty acids than with carbohydrates and amino acids.

It seems likely that the energy generated in the photo-oxidation of these organic substances supplies a part of the energy necessary for the photoformation of formaldehyde. We are of the opinion that in nature, the photosynthesis that is taking place in the plants is aided by the energy obtained in plant respiration, which goes on as long as the plant lives. The ease with which formaldehyde or other energy-rich compounds are formed in plants is partly due to their getting a constant supply of energy from the oxidation of the food materials present in the plant. We have postulated that the most important chemical change in the formation of carbohydrates in plants and in the formation of formaldehyde in nature from carbon dioxide and water is the photolysis of water into H and OH. The amount of energy required to decompose a gram molecule of water into H and OH is approximately the same as that necessary for the formation of a gram mole of formaldehyde from carbon dioxide and water. These are highly endothermal changes requiring radiations of wavelength 2552Å (112 Calories). In nature, however, photosynthesis takes place in visible light especially the red. We are of the opinion that the energy derived from respiration in the plants already supplies a part of the energy necessary for the photosynthesis and thus renders the photodecomposition of water possible by longer wavelengths. Although the adsorption of carbon dioxide and water by the chlorophyll of the leaf may partially activate these substances just as the adsorption of hydrogen and oxygen on a platinum or palladium surface renders them active, it appears to us that this activation of carbon dioxide and water by their adsorption on the leaf surface is less important than their activation by the adsorption of energy from respiration.

There is an intimate relation between respiration and photosynthesis in the plant kingdom, because photosynthesis proceeds with difficulty without the energy available from respiration for the partial activation of carbon dioxide and water vapour. The need of the presence of oxygen in photosynthesis is also explained from the same point of view.

It is easier to obtain formaldehyde or any other energy-rich compound from carbonic acid and bicarbonate solution on exposure to light when a suitable exothermal reaction is taking place in the system along with the photosynthetic reaction.

It is well known that large amounts of energy are liberated in the oxidation of carbohydrates

$$C_6H_{12}O_6 + 6O_2 = 6CO_2 + 6H_2O + 676 \text{ Cal.}$$

This energy can be utilised in bringing about the endothermal reaction between nitrogen and oxygen ($N_2 + O_2 + 43{\cdot}2 = 2\ NO$) on the soil surface even in absence of sunlight. The oxide of nitrogen in contact with the soil moisture and lime can cause the formation calcium nitrate, which may be reduced to ammonium salts by the excess of carbohydrates added to the soil. Moreover the energy liberated in the oxidation of carbohydrates on the soil surface may lead to the decomposition of water molecules according to the following equation :—

$$H_2O + 112\ Cal = H + OH.$$

The atomic hydrogen can readily combine with molecular nitrogen forming ammonium salts on the soil surface. When soil mixed with carbohydrates, glycerol, etc., comes in contact with air, the carbon decreases and the total and ammoniacal nitrogen increase by nitrogen fixation. By light absorption, all these processes, which can take place in the dark provided the energy of the oxidation of the carbohydrates is available on the soil surface, are accelerated and hence more nitrogen fixation per gram of energy material oxidized is observed in light. Under natural conditions large amounts of nitrogen are fixed in the soil with the energy of the oxidation of celluloses aided by sunlight.

According to Moore[9] (1921) sunlight causes a slight union of nitrogen and oxygen in the air, resulting in the formation of oxides of nitrogen. Dhar and Sanyal[7] (1925) have observed the formation of traces of nitrous acid when air freed from impurities is bubbled through conductivity water in presence of ultraviolet light. This reaction cannot go far, as nitrous acid decomposes readily in light according to the equation

$$3\,HNO_2 \rightleftarrows HNO_3 + 2\,NO + H_2O$$

Moreover nitric acid and nitrates also decompose in light. Similarly the following reaction $CO_2 + H_2O + 112$ Cal. $= HCHO + O_2$ can take place to a very limited extent in air as formaldehyde is readily decomposed in light. Dhar and Atma Ram obtained 0·001 gram formaldehyde per litre of freshly collected rain water. The combinations of nitrogen and oxygen and carbon dioxide and water vapour are greatly accelerated when, along with sunlight, the energy of the carbohydrate oxidation is available to the systems.

The efficiency of nitrogen fixation is also low as in the case of carbon assimilation. It is well known that in many cases, only 0·1 to 0·2 % of the light energy is utilized in carbon assimilation in plants. In our experiments with nitrogen fixation, not even 1% of the energy liberated by the oxidation of the carbohydrates or cellulosic materials is utilized in fixation.

N.B.—As the publication of the paper was delayed for a long time, a few minor additions and alterations had to be made.

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NITROGEN FIXATION AND AZOTOBACTER COUNT ON THE APPLICATION OF CARBOHYDRATES AND OTHER ENERGY-RICH MATERIALS TO THE SOIL, PART III

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Received February 24, 1937

SUMMARY

1. The influence of sunlight on nitrogen fixation and Azotobacter count in the soils mixed with energy-rich materials has been studied.

2. Nitrogen fixation is always greater in the exposed soils than in the soils kept in the dark.

3. When calculated the nitrogen fixed per gram of carbon oxidised is practically double in the exposed soils than in the dark.

4. Light plays an important part in the nitrogen fixation in the soils when supplied with energy materials.

5. Even in the field soils nitrogen fixation takes place on the addition of starch and glycerol.

In a previous paper[2] it has been reported that the nitrogen fixation with soils containing carbohydrates like canesugar and glucose in basins is always greater when exposed to sunlight daily for about 8 hours than in basins kept in dark although the Azotobacter count is very much greater in the dark basins than in the light.

I have extended my experiments to mannitol, dextrin, maltose, fructose and galactose as energy materials. The experimental procedure is the same as recorded in the Part II.

In order to see whether nitrogen fixation takes place in field conditions I have applied definite amounts of starch and glycerol to small plots of land.

Three plots of equal area (4′ by 4′) were taken. To one plot 2 kilograms of glycerol and to the other 2 kilograms of starch were applied. Another plot was kept as control. All these plots were dug up and watered regularly to keep an uniform moisture content. At regular intervals the Azotobacter count and nitrogen estimations of these plots were carried on.

The following are the results obtained :—

Table I

1 kilogram of soil + 20 gms. Mannitol exposed (Temp. 38°—48°)

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. of dry soil	
20-4-1936 original soil	0·00152	0·043	0·494	6·5	Nitric nitrogen varied from 0·00308 % to 0·00358 %.
5-5-1936	0·002	0·043	1·2087	11·8	
22-5-1936	0·0028	0·0454	1·0725	16·6	
8-6-1936	0·0035	0·0473	0·9539	20·5	
24-6-1936	0·00436	0·0489	0·8118	28·5	
14-7-1936	0·005	0·0507	0·7156	32·5	
26-7-1936	0·00536	0·051	0·6432	29·8	
10-8-1936	0·0052	0·051	0·5846	30·5	
12-10-1936	0·00424	0·0507	0·5124	18·5	

Table II

1 kilogram of soil + 20 gms. Mannitol dark (Temp. 30°—38°)

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. of dry soil	
20-4-1936 original soil	0·00152	0·043	0·494	6·5	Nitric nitrogen varied from 0·00308 % to 0·0032 %
5-5-1936	0·00174	0·043	1·2266	17·5	
22-5-1936	0·00207	0·043	1·1654	29·2	
8-6-1936	0·0025	0·0454	1·0912	110·0	
24-6-1936	0·00294	0·0454	1·0091	175·0	
14-7-1936	0·00341	0·0466	0·9299	235·0	
26-7-1936	0·00368	0·0473	0·8516	292·6	
10-8-1936	0·00384	0·0477	0·7634	355·8	
12-10-1936	0·00341	0·048	0·5106	315·5	

Table III

1 kilogram of soil + 20 gms. Dextrin exposed (Temp. 34°—42°)

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. of dry soil	
8-10-1936 original soil	0·00145	0·057	0·6156	5·2	Nitric nitrogen varied from 0·0032% to 0·00376%
28-10-1936	0·00152	0 057	1·3564	6·1	
17-11-1936	0·0021	0·059	1·2772	12·6	
10-12-1936	0·0028	0·0608	1·1926	17·2	
28-12-1936	0·00364	0·062	1·0742	20·5	
18-1-1937	0·004	0·0636	0·9414	28·5	
4-2-1937	0·0038	0·0646	0·7728	20·5	
20-2-1937	0·00336	0·064	0·6292	18·5	
6-3-1937	0·00304	0·0636	0·6086	11·5	
28-3-1937	0·00258	0·0626	0·5984	8·5	

Table IV

1 kilogram of soil + 20 gms. Dextrin dark (Temp. 28°—31°)

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. of dry soil	
8-10-1936 original soil	0·00145	0·057	0·6156	5·2	Nitric nitrogen varied from 0·0032% to 0·0031%
28-10-1936	0·00145	0·057	1·3892	4·8	
17-11-1936	0·0016	0·057	1·3258	18·7	
10-12-1936	0·00188	0·0586	1·2644	32·5	
28-12-1936	0·0021	0·059	1·1916	82·5	
18-1-1937	0·00224	0·06	1·1032	150·5	
4-2-1937	0·00228	0·0604	0·9778	198·5	
20-2-1937	0·00224	0·0608	0·8454	225·0	
6-3-1937	0·00204	0·0612	0·6868	280·0	
28-3-1937	0·00196	0·0608	0·6648	205·0	

Table V

1 kilogram of soil + 20 gms. Maltose exposed (Temp. 34°-42°)

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. of dry soil	
8-10-1936 original soil	0·00145	0·057	0·6156	5·2	Nitric nitrogen varied from 0·0032 % to 0·00388 %
26-10-1936	0·0016	0·057	1·3684	4·2	
12-11-1936	0·00215	0·059	1·2672	16·2	
1-12-1936	0·0027	0·0604	1·1694	21·5	
17-12-1936	0·00322	0·0618	1·0712	32·8	
31-12-1936	0·004	0·0626	0·9658	30·2	
19-1-1937	0·0044	0·064	0·8334	31·5	
5-2-1937	0·00388	0·0656	0·6868	25·5	
22-2-1937	0·0032	0·064	0·6426	16·5	
7-3-1937	0·0028	0·0636	0·6284	12·5	
29-3-1937	0·0024	0·0626	0·6168	7·5	

Table VI

1 kilogram of soil + 20 gms. Maltose dark (Temp. 28° − 31°)

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. of dry soil	
8-10-1936 original soil	0·00145	0·057	0·6156	5·2	Nitric nitrogen varied from 0·0032 % to 0·0031 %
26-10-1936	0·00148	0·057	1·3944	4·95	
12-11-1936	0·00168	0·057	1·3163	21·2	
1-12-1936	0·0019	0·0586	1·2451	40·5	
17-12-1936	0·0021	0·0596	1·1699	76·5	
31-12-1936	0·00256	0·0608	1·0873	125·6	
19-1-1937	0·0028	0·0612	0·9958	202·5	
5-2-1937	0·00284	0·0612	0·9062	215·0	
22-2-1937	0·0028	0·0612	0·7936	230·0	
7-3-1937	0·0024	0·0618	0·6652	250·0	
29-3-1937	0·00208	0·0612	0·6426	210·0	

Table VII

1 kilogram of soil + 20 gms. Fructose exposed (Temp. 34°–42°)

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. of dry soil.	
8-10-1936 original soil	0·00145	0·057	0·6156	5·2	Nitric nitrogen varied from 0·0032 % to 0·00372 %
25-10-1936	0·00168	0·057	1·3568	6·1	
11-11-1936	0·0021	0·0586	1·2622	13·2	
1-12-1936	0·0028	0·0608	1·1518	19·8	
17-12-1936	0·00336	0·0622	1·0264	36·0	
31-12-1936	0·00428	0·0636	0·8978	34·5	
19-1-1937	0·00452	0·0646	0·7614	29·8	
5-2-1937	0·0042	0·0656	0·6346	23·5	
22-2-1937	0·0038	0·0646	0·6284	15·0	
7-3-1937	0·00304	0·0636	0·6126	10·5	
29-3-1937	0·00271	0·0626	0·6028	7·0	

Table VIII

1 kilogram of soil + 20 gms. Fructose dark (Temp. 28°–31°)

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. of dry soil	
8-10-1936 original soil	0·00145	0·057	0·6156	5·2	Nitric nitrogen is 0·0032 %.
25-10-1936	0·00152	0·057	1·3745	7·2	
11-11-1936	0·00168	0·057	1·3161	19·8	
1-12-1936	0·00186	0·059	1·2429	35·8	
17-12-1936	0·00215	0·06	1·1633	82·5	
31-12-1936	0·00252	0·0608	1·0818	136·5	
19-1-1937	0·0028	0·0612	0·9876	225·5	
5-2-1937	0·00252	0·0618	0·8434	260·0	
22-2-1937	0·00248	0·0618	0·7318	275·0	
7-3-1937	0·0024	0·0622	0·6102	290·0	
29-3-1937	0·00224	0·0612	0·6028	210·0	

Table IX

1 kilogram of soil + 20 gms. Galactose exposed (Temp. 34° − 42°)

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. of dry soil	
19-10-1936 original soil	0·00145	0·057	0·6156	5·6	Nitric nitrogen varied from 0·0032 % to 0·004 %
10-11-1936	0·00176	0·057	1·3432	9·2	
26-11-1936	0·00224	0·06	1·2386	17·8	
10-12-1936	0·00289	0·0613	1·1128	19·2	
28-12-1936	0·00336	0·062	0·9802	22·5	
18-1-1937	0·004	0·064	0·8468	24·2	
4-2-1937	0·00388	0·065	0·7036	22·5	
20-2-1937	0·00336	0·066	0·5986	18·5	
6-3-37	0·0027	0·065	0·5628	10·5	
28-3-1937	0·0024	0·0636	0·5584	8·0	

Table X

1 kilogram of soil + 20 gms. Galactose dark (28° − 31°)

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. dry soil	
19-10-1936 original soil	0·00145	0·057	0·6156	5·6	Nitric nitrogen varied from 0·0032 % to 0·00336 %
10-11-1936	0·00152	0·057	1·3618	13·6	
26-11-1936	0·00172	0·057	1·2904	26·8	
10-12-1936	0·0019	0·059	1·2092	38·6	
28-12-1936	0·0021	0·06	1·1176	92·8	
18-1-1937	0·00252	0·0604	1·0178	195·5	
4-2-1937	0·00256	0·0608	0·8942	225·0	
20-2-1937	0·00268	0·0618	0·7456	295·0	
6-3-1937	0·0024	0·0618	0·6492	245·0	
28-3-1937	0·0021	0·0612	0·6284	215·0	

Table XI

Plot 4′ × 4′ containing 2 kilograms of Glycerol

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. of dry soil	
12-10-1936 original soil	0·0009	0·042	0·4603	1·95	Nitric nitrogen varied from 0·002 % to 0·003 %
30-10-1936	0·0012	0·042	0·7973	2·8	
18-11-1936	0·00186	0·0437	0·7317	9·5	
5-12-1936	0·0021	0·0451	0·6702	20·6	
25-12-1936	0·0028	0·0466	0·5878	23·5	
18-1-1937	0·00197	0·0472	0·4954	17·5	
3-2-1937	0·0019	0·0472	0·4528	15·0	
23-2-1937	0·0014	0·0466	0·4468	10·0	
22-3-1937	0·00134	0·0461	0·4356	6·5	

Table XII

Plot 4′ × 4′ containing 2 kilograms of Starch

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. of soil	
12-10-1936 original soil	0·00087	0·0466	0·5033	2·2	Nitric nitrogen varied from 0·00172 % to 0·0024 %
30-10-1936	0·00124	0·0466	0·9213	3·1	
18-11-1936	0·00174	0·0482	0·8731	8·8	
5-12-1936	0·002	0·05	0·8205	19·8	
25-12-1936	0·00268	0·0512	0·7442	20·5	
18-1-1937	0·002	0·0525	0·6456	16·5	
3-2-1937	0·00197	0·0531	0·5604	14·5	
23-2-1937	0·00174	0·0525	0·5518	9·5	
22-3-1937	0·0014	0·0518	0·5384	5·5	

Table XIII

Control Plot 4′×4′

Date	Ammoniacal nitrogen %	Total nitrogen %	Total carbon %	Number of Azotobacter in millions per 1 gm. of soil	
12-10-1936	0·00116	0·054	0·567	2·47	Nitric nitrogen 0·002 % is constant
30-10-1936	0·00116	0·054	0·567	2·52	
18-11-1936	0·00116	0·054	0·567	2·01	
5-12-1936	0·00116	0·054	0·567	2·92	
25-12-1936	0·00116	0·054	0·567	2·22	
18-1-1937	0·00116	0·0538	0·5328	1·95	
3-2-1937	0·00116	0·0538	0·5286	2·5	
23-2-1937	0·00105	0·0538	0·5286	2·05	
22-3-1937	0·00105	0·0538	0·5248	2·5	

Discussion

The foregoing results show that although in the dark the Azotobacter numbers are approximately ten times than in the dishes exposed to sunlight, the nitrogen fixation in dishes receiving sunlight is almost double than in those kept in dark. Moreover a far-reaching conclusion can be drawn from the above experimental results when the amount of nitrogen fixed per gram of carbon oxidised with various energy-rich materials is calculated. The following table shows the amount of nitrogen fixed per gram of carbon oxidised both in light and dark.

Table XIV

	Nitrogen fixed	
Substance	Light	Dark
Mannitol	12·82 mgm.	6·98 mgm.
Dextrin	13·0	5·98
Maltose	12·6	6·58
Fructose	11·91	6·8
Galactose	12·08	6·73

The above results show conclusively that for the same amount of carbon oxidised of the energy materials the nitrogen fixation in light is practically double than that in the dark. Dhar and Mukerji[1] have explained that the formation of

nitric oxide may be the first step in the fixation of nitrogen. It is well known that the combination of nitrogen and oxygen forming nitric oxide according to the equation

$$N_2 + O_2 + 43{\cdot}2 \text{ Cal} \rightarrow 2 \text{ NO}$$

is an endothermal chemical change which should take place when supplied with required energy. When the energy materials are mixed with soil they undergo oxidation as is evident from the decrease of carbon with time. A gram molecular weight of these energy materials in their oxidation liberate energy much in excess of that required for the combination of nitrogen and oxygen forming nitric oxide. Even in the dark nitrogen fixation takes place in soils when supplied with energy materials but the efficiency of this process is very low. When, however, the soils containing the energy materials are exposed to light daily for eight hours, not only the velocity of oxidation of the energy materials will increase as is evident from the quicker disappearance of carbon in light than in dark but the efficiency of the process is also considerably increased. It seems therefore that the nitrogen fixation which is an endothermal reaction, is considerably aided not only by the energy obtained from the oxidation of the carbohydrates but also by the absorption of the incident light. As there is an intimate contact between nitrogen and oxygen on the soil surface, which is receiving energy both from the sun and from the oxidation processes, it is quite reasonable to expect more nitrogen fixation in light than in dark and this is actually observed experimentally. Hence light actually takes part in the nitrogen fixation in the soils when supplied with the energy materials.

In the field trials with starch and glycerol I find the same behaviour as in dishes, the carbon decreases and the nitrogen increases. Hence in fields when the energy materials like glycerol and starch undergo oxidation, nitrogen is fixed in the soil. No change either in the total nitrogen or Azotobacter numbers is observed in the control plot.

My thanks are due to the Imperial Council of Agricultural Research for a grant which enabled me to do the above work.

I am also indebted to Prof. N. R. Dhar, D.Sc., for his kind suggestion and keen interest in this work.

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CHANGES IN SOIL NITROGEN AFTER THE ADDITION OF FRESH COWDUNG TO SOIL

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Communicated by Prof. N. R. Dhar

Received February 26, 1937

SUMMARY

1. Fresh cowdung has been shown to fix atmospheric nitrogen in dishes, in fields and in sterile vessels exposed to sunlight.

2. The amount of nitrogen fixed as well as the amount of nitrogen fixed per gram of carbon oxidised is greater in light than in the dark.

3. Molasses seems to stimulate the oxidation of cowdung in the soil, and increases the total amount of nitrogen fixed.

4. Under the conditions of these experiments no white ants were seen to develope in the soil after the treatment with cowdung.

The following lines from Waksman's *Principles of Soil Microbiology* 1932, pp. 491, show that different results have been obtained by different workers regarding the fixation of nitrogen on the addition of cellulosic matters to the soil, "Polysaccharides, like cellulose, can also serve as valuable sources of energy if they are first partially broken down by cellulosic decomposing organisms [1,5,6,7,8]. These results need further confirmation; this process is of particular importance, since the bulk of the energy material commonly added to the soil consists of cellulose and pentosans" and incontestable proof of nitrogen fixation in the soil is still lacking. There are indications in the literature that certain hemicelluloses, including pentosans, can be utilised as direct source of energy in nitrogen fixation.

The object of the present investigation is to find out whether cowdung which consists mainly of pentosans and cellulosic materials together with a small percentage of protein nitrogen, when added to the soil can bring about nitrogen fixation in the soil.

From time immemorial cowdung is used as a fertiliser in the agricultural practice in India and in other parts of the world. In India the farmers generally store the animal dung in pits near their houses and transfer to the fields two to three months before the sowing season.

Various methods of composting have been put forward to make cowdung suitable for crops. The general belief is that fresh cowdung will produce harmful effect on the crops. The main principle of all these processes is to get rid of most of the carbonaceous matters, which are thought to exert a bad effect on plant growth. Thus only a part of the total nitrogen originally present in the cowdung together with its whole mineral content is transferred to the fields. The whole of the easily oxidisable carbonaceous substances and some cellulosic matters are destroyed by anaerobic micro-organisms and the residue contains less organic matter than before.

In recent publications from this laboratory [2,3] it has been shown that energy-rich materials when added to soil stimulate nitrogen fixing factors and considerable amount of nitrogen fixation takes place in the soil. Therefore if cowdung is added directly to soil a part of the energy derived from the oxidation of the organic materials present in cowdung may be utilised to enrich the soil nitrogen, besides other ingredients already present in cowdung.

The following are the carbon and nitrogen content of a few samples of fresh cowdung :—

Table I

No.	NITROGEN	CARBON	C/N	No.	CARBON	NITROGEN	C/N
1.	0·3684%	7·499%	20·3	4.	8·194%	0·3189%	25·7
2.	0·358	9·487	27·5	5.	11·68	0·3783	32·3
3.	0·2626	12·409	21·5	6.	6·841	0·378.	18·1

These results show that the carbon-nitrogen ratio in cowdung is greater than 20 and nitrogen is low; therefore large amounts of dung manure is generally added to the fields for a high crop yield. Moreover in the method of composting the organic carbon is practically destroyed and the finished product containing 1 to 2 per cent nitrogen when applied to the fields is attended by losses of nitrogen in gaseous form.[4]

The loss of nitrogen by this process may amount to two to three times that taken up by a crop. Therefore we must mould our agricultural practices in such a way as to utilise the whole of the nitrogen added, prevent the nitrogen loss and stimulate nitrogen fixing factors in the soil. This problem of nitrogen changes in the soil after an application of cowdung needs a thorough scientific investigation for a clear elucidation of the chemical processes, which are of high economic and scientific interest.

Accordingly, experiments were undertaken to study the nitrogen changes which take place in soil and in dishes after treating with cowdung. The effect of sunlight on these reactions has also to be investigated. The following are the experimental results obtained :—

Table II

Soil=500 gms.+cowdung=100 gms. Started on 28-7-1936.

Condition.	NH_3-N,	NO_3-N,	Total−N,	Total−C.	Date of Sampling
Continued from Presidential Address, National Acad. Sci. (India) 1937, pp. 30.					
Light.	0·002%	0·0036%	0·1308%	1·1068%	12-1-1937
Shade.	0·001	0·0022	0·1288	1·0467	12-1-1937
Light.	0·0024	0·0042	0·0986	0·9244	24-4-1937
Shade.	0·0012	0·0018	0·1186	1·00	24-4-1937

Table III

Soil=500 gms.+cowdung=100 gms.+molasses 5 gms. Started on 28-7-1936.

Continued from Presidential Address, National Acad. Sci. (India) 1937, pp. 30.					
Light.	0·0024	0·003	0·1378	1·1161	12-1-1937
Shade.	0·0017	0·0022	0·1288	0·9542	12-1-1937
Light.	0·0018	0·0037	0·1012	1·042	24-4-1937
Shade.	0·0014	0·0018	0·1082	0·8642	24-4-1937

These results show that nitrogen fixation takes place when cowdung is mixed with soil and is kept under natural conditions. These observations were expected to hold good under field conditions. Field experiments were performed to verify these observations under field conditions. The following are the results obtained.

Table IV

Plot area=36 sq. ft. Cowdung 10 kg. Started on 20-11-1936.

NH_3-N,	NO_3-N,	Total−N,	Total-C	Date of Sampling.
0·0017%	0·002%	0·0583%	0·6899%	20-11-1936
0 0017	0·002	0·0626	0·5859	23-12-1936
0·0014	0·0024	0·0631	0·5224	19-1-1937
0·0012	0·0025	0·0667	0·4286	19-2-1937
0·0013	0·0026	0·0609	0·4312	17-3-1937
0·0014	0·0029	0·0544	0·4312	4-5-1937

Table V

Plot area=36 sq. ft. Cowdung=30 kg.+Molasses=30 kg. Started on 20-11-1936.

0·0021	0·0028	0·09	2·184	20-11-1936
0·0043	0·0028	0·0934	1·505	23-12-1936
0·002	0·0028	0·10	1·138	19-1-1937
0·0022	0·0028	0·1024	0·8214	19-2-1937
0·0015	0·0030	0·0823	0·8285	17-3-1937
0·0014	0·0032	0·0812	0·8124	4-5-1937

Table VI

Plot area=36 sq. ft. Cowdung 60 kg.+Molasses 60 kg. Started on 20-11-1936.

0·004	0·0046	0·14	4·035	20-11-1936
0·0023	0·0046	0·14	3·753	23-12-1936
0·0028	0·0031	0·14	2·511	19-1-1937
0·0030	0·0031	0·129	1·3424	19-2-1937
0·002	0·0031	0·1273	0·8614	17-3-1937
0·0013	0·0037	0·1013	0·8444	4-5-1937

In recent publications (*loc. cit.*) sunlight has been shown to play an important part in the soil processes. Experiments were undertaken to verify these conclusions under field conditions. On this account similarly treated fields were covered with wooden planks to avoid direct exposure to sunlight and some were exposed to sunlight. No watering was done and the plots were ploughed one evening in each month. The following are the experimental results.

Table VII

Plot area=36 sq. ft. Cowdung=30 kg. Exposed to sunlight.

NH_3-N,	NO_3-N,	Total−N,	Total−C,	Date of Sampling.
0·0018%	0·0018%	0·0636%	0·8801%	20-11-1936
0·0015	0·002	0·0742	0·6921	19-1-1937
0·0013	0·0022	0·0751	0·5212	19-2-1937
0·0012	0·0024	0·0714	0·555	17-3-1937
0·0021	0·0031	0·0651	0·535	4-5-1937

Table VIII

Covered with wooden planks.

0·0015	0·0018	0·0584	0·8453	20-11-1936
0·0009	0·0018	0·0583	0·644	19-1-1937
0·0008	0·0019	0·0592	0·5821	19-2-1937
0·0008	0·0018	0·0538	0·6163	17-3-1937
0·0006	0·0014	0·0533	0·6012	4-5-1937

Table IX

Plot area=36 sq. ft. Cowdung 19·5 kg. Exposed to sunlight.

0·0017	0·0025	0·0692	0·8616	20-11-1936
0·0014	0·0028	0·0768	0·6112	19-1-1937
0·0012	0·0028	0·0774	0·5001	19-2-1937
0·0008	0·0029	0·0636	0·5306	17-3-1937
0·0014	0·0038	0·0682	0·534	4-5-1937

Table X

Covered with wooden planks.

0·0017	0·0026	0·07	0·8692	20-11-1936
0·0011	0·0018	0·0731	0·6538	19-1-1937
0·0011	0·0018	0·0738	0·5112	19-2-1937
0·0007	0·0018	0·07	0·5692	17-3-1937
0·0005	0·0012	0·0694	0·5342	4-5-1937

Table XI

Plot area = 36 sq. ft. Cowdung 60 kg. Exposed to sunlight.

NH_3-N,	NO_3-N,	Total−N,	Total−C,	Date of Sampling.
0·0018 %	0·0018 %	0·0744 %	0·9703 %	20-11-1936
0·0017	0·0025	0·0832	0·8212	19-1-1937
0·0014	0·0026	0·0841	0·6112	19-2-1937
0·0015	0·0028	0·0778	0·6812	17-3-1937
0·0028	0·0031	0·0712	0·6424	4-5-1937

Table XII

Covered with wooden planks

0·0025	0·0031	0·0875	1·672	20-11-1936
0·0013	0·0022	0·0862	0·937	19-1-1937
0·001	0·0018	0·0854	0·8114	19-2-1937
0·0009	0·0018	0·0812	0·6277	17-3-1937
0·0007	0·0011	0·08	0·6172	4-5-1937

These results indicate that cowdung along with the cellulosic materials when turned into soil under Indian conditions is more beneficial than that of composting. The treated fields show a greater percentage of nitrogen which goes on increasing till most of the organic matter is oxidised and then it slightly decreases due to the loss of nitrogen in the process of nitrification. The evolution of carbondioxide which is enhanced by the addition of organic matter brings some of the calcium, iron, phosphates and magnesium carbonates into solution in the form of mostly bi-carbonates. These soluble salts enhance the microbiological and oxidation processes in the soil.

Another important point has been brought out in this work is that the process of fixation of atmospheric nitrogen apart from the well-known bacterial action is also partly photochemical, catalytic and thermal process. The fields covered with wooden planks show less fixation per gram of carbon oxidised than those plots receiving direct sunlight. Similar results have been obtained with soils in dishes.

To study the oxidation and fixation of atmospheric nitrogen by cowdung under perfectly sterilised conditions the following arrangements were made. Pure air completely freed from ammonia, nitric oxide, carbondioxide and bacteria by passing through solutions of ferrous sulphate and sulphuric acid, concentrated sulphuric acid and baryta is bubbled through a sterile suspension of cowdung and water in a quartz flask. The air then passes through a solution of standard baryta to catch the carbondioxide and then another flask containing dilute sulphuric acid to catch ammonia. Two more flasks containing baryta and dilute sulphuric acid are joined in series to prevent the incidence of any ammonia or carbondioxide from outside.

A similar experiment was performed under identical conditions except that the reaction vessel was that of ordinary glass and wrapped thoroughly by a black cloth to cut off light. Sterilisation of the suspension, container and the connections were done in an autoclave at 15 lbs. pressure for three hours. To test for complete sterilisation a control emulsion was similarly treated at the same time and plated for Azotobacter and total bacteria. No colony was seen to develop even after ten days' incubation. The following are the results obtained after 100 hours' exposures to sunlight.

Original carbon in the mixture of soil and cowdung		0·9372 %
„ Nitrogen „ „		0·05935 %
Total nitrogen left in the exposed vessel	(*a*)	0·06034 %
	(*b*)	0·06087
	(*c*)	0·06034
	(*d*)	0·05932
Total carbon in the exposed vessel	(*a*)	0·837 %
	(*b*)	0·8375
	(*c*)	0·837
	(*d*)	0·837

Total nitrogen in the vessel kept in the sunlight wrapped in a black cloth.	(*a*)	0·05931%
	(*b*)	0·05908
Total carbon left in the vessel	(*a*)	0 9207
	(*b*)	0·9207

Mean carbon oxidised in the light :—0·1002 gm.
Mean nitrogen fixed in the light :—0·86 ml. gm.
Mean carbon oxidised in the dark :—0·0165 gm.
Mean nitrogen fixed in the dark :—nil

This as well as the dish and the field experiments demonstrate clearly that not only the amount of nitrogen fixation is greater in sunlight, but the amount of

nitrogen fixed per gram of carbon oxidised is also greater in light. Hence it is assumed that sunlight takes a direct part in the process of nitrogen fixation from the energy derived from the oxidation of the energy-rich materials in soil.

Thanks are due to Prof. N. R. Dhar, D.Sc., I.E.S., etc., for his kind guidance and interest in this work.

N.B.—As the publication of the paper was delayed for a long time a few minor additions had to be made.

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